Female Labour Force Participation in an Era of Organizational and Technological Change

Marina Adshade
Queen's University

Abstract

This paper examines the endogenous interaction between the rise in female labour force participation and changes in both the method and mode of production that occurred during the early part of the 20th century. Within a dynamic general equilibrium framework, an exogenous expansion in the skill level of the population induces an organizational change at the firm level and a redirection of investment towards new technologies that complement the skills of an emerging workforce. As such, an increase in the potential supply of skilled workers leads to an increase in the demand for skilled, female, workers and an overall increase in female labour force participation. In addition to allowing for a change in the mode of production in a market with directed technical change, this model establishes a framework in which to explicitly examine the transitional dynamics as the supply of skilled workers becomes relatively abundant. Whereas earlier theories fail to explain the movement in women's wages in this period; both the increase in women's wages and the decline in clerical wages relative to manufacturing can be explained by this model.

1 Introduction

Prior to 1890 few women in the United States were employed in the waged workforce. At that time, more than 95% of married women were employed exclusively in the home, producing goods and services for the household (Goldin,1990).[1] Caring for the

[0]I am very grateful to Huw Lloyd-Ellis for his assistance on this project and to Shannon Seitz, Ian Keay, Alan Green, Mary MacKinnon, Alan Green, Carolyn Moehling, Beverly Lapham for their helpful comments. Financial support for this project was provided by TARGET.

[1]It is widely accepted that this figure, reported by the 1890 Census, underestimates the employment of women. The census excluded women who worked as unpaid workers on family farms and in family businesses, as well as women who produced goods in the home for market and ran boardinghouses (see Goldin (1990) and Rotella (1981) for more detail). It is, however, a reasonable approximation of the employment of women in the formal waged workforce at the time.

needs of the average family entailed 58 hours per week of labour input (Lebergott 1993). Sparsely available public schools meant that few individuals, male or female, had the opportunity to obtain formal schooling above the elementary level, so that high school graduation rates were only 3.5% in 1889 (U.S. Department of Education).[2] The turn of the century marked a significant transition for women in terms of education and employment. Secondary school attendance and graduation rates began rising steadily at the end of the 19th century. By 1940, 50% of all 17 year olds were graduating from high school, with the number female graduates exceeding male graduates by about 2%. By that same year, 45.5% of single women and 13.8% of married women were formally employed. This early transition of women away from home production and into formal waged employment was the beginning of a remarkable trend that constituted the single largest change in the labour force in the twentieth century.

Recent literature has revived the debate on the basis of this movement of women away from home production and into waged employment. Earlier work (Mincer, 1962; Smith and Ward, 1984) concludes that women increased their labour supply in response to increases in wages paid to female workers (labour demand increased while labour supply remained relatively stable and inelastic). Goldin (1990) reconsiders Mincer's analysis, allowing parameter estimates to change over the period, and concludes that supply side effects outweighed demand side effects, citing increasing education and declining fertility rates as the main contributors. Rotella (1981) argues that the adoption of new technologies reduced the skill-specific requirement for clerical jobs and led to an increase in demand for female workers. More recently Greenwood, Seshadri and Yorukoglu (2001) conclude that increases in the female labour supply can be attributed directly to the falling price and increased availability of substitutes to home production in the form of durable goods.

This paper considers an alternative explanation. As education levels increased at the end of the 19th century, firm structure changed in order to benefit from the growing number of skilled workers.[3] Tasks at the managerial level became more specialized and work that had been previously undertaken by a few educated managers was now undertaken by many skilled clerical workers. Growth in employment in this sector was accompanied by an acceleration in the adoption of technologies used by clerical workers. As male workers held the comparative advantage in manual labour, new clerical positions were occupied predominantly by educated, female, workers. Within this framework, the demand for female workers increased due to both a change in the

[2]High school graduation rates are calculated as the percentage of 17 year olds in the population who have completed their high school education. Data provided by the US Bureau of the Census, *Current Population Reports*, Series P-20, *Educational Attainment in the United States*.

[3]Where workers are considered 'skilled' if they possess advanced literacy and numeracy skills. While these workers would by no means be considered skilled by modern standards, given the level of education in this period they are skilled relative to the average worker.

method of production (i.e. the organizational structure of the firm) and the increased adoption of technologies that were directed towards the increasing supply of educated women.

The rate of enrollment and graduation from American high schools grew rapidly over the last quarter of the 19th and first half of the 20th centuries. While the fastest rates of growth for high school graduation were seen in the decades between 1910 and 1940, the foundation for this growth began much earlier. Growth in public school enrollment rates began as early as the 1860's, with a 44% increase in the 1870's (U.S. Department of Education, 1993). While high school graduation rates in 1910 were significantly lower than in 1940 (8% versus 50%) and high school degrees still very much the exception rather than the rule, the increases in high school graduation rates over the period 1870 to 1910 were still considerable, with a five-fold increase over the period. Many factors have been attributed to influencing rising educational levels; the declining cost of transportation, increasing agricultural productivity, legislation prohibiting child labour, curriculum changes as well as increased corporate demand for literate, and numerate, workers.[4]

Increases in the demand for clerical workers may have been responsible for increases in education levels, as the wage premium paid to educated workers increased. There are several reasons to believe that the causation runs in the other direction, however. Firstly, the foundation for large scale high school enrollment preceded the large increases in graduation *rates* seen in the 1920's and 1930's, and appears to have begun as early as 1870, with large increases in public school enrollment. Secondly, while the number of female graduates is marginally higher in every decade than male graduates, rapid growth in the number of male high school graduates cannot be supported by increasing demand for clerical workers, as most of this demand was filled by female workers. While electrification and innovation in manufacturing could have created an excess demand for skilled male workers it is equally plausible that the causation runs in the opposite direction: increasing availability of skilled workers, both male and female, led to increases in the adoption of technologies that utilized the skills that those workers made available and, in the case of the clerical workers, a restructuring of the firm to incorporate those skills into the production technology.

The period 1890 to 1940 was also one of significant change in the nature of production. Prior to this period, jobs could be divided roughly into two categories: manual labour and managerial labour. The majority of jobs fell into the first category and were performed primarily by men with little or no education.[5] The second category of jobs was performed by a small number of educated men who were wholly responsible for the running of the firm. During this period, firms began to undertake a

[4]For more information on the expansion of high school enrollment and graduation rates in the US see Goldin (1999).

[5]For example, in 1890 the iron and steel industry in the US employed 147,357 workers, only two of which were women. (Goldin, 2002)

division of labour at the administrative level; soon many firms had a few managers, who were ultimately responsible for the operations of the business, and many clerical workers who performed the day-to-day tasks. These clerical jobs occupied the middle ground of employment, requiring a relatively high level of education but none of the specialized training of highly skilled jobs or the physical endurance of unskilled jobs. The share of female workers employed in this rapidly growing clerical sector grew from 4% in 1890 to 21% in 1930. By 1930, 50% of all clerical workers were female, up from 15% in 1890 (Goldin, 1990). Women had a comparative advantage in clerical labour relative to manual labour: Goldin and Katz (1999, table 2) compute the ratio of earnings of individuals in clerical positions to those in manufacturing and find the relative earnings of female clerical workers to female manufacturing workers were significantly higher than those of male clerical workers to male manufacturing workers. In 1923, female clerical workers earned approximately 1.5 times more than their female counterparts in production whereas male clerical workers earned the same wage as male production workers.

This period is also notable for the rapid adoption of office technologies. Typewriters, duplicators and bookkeeping machines started to become commonplace by 1910. Most of these new technologies required the general literacy and numeracy skills of the new high school graduates. At the same time new manufacturing processes, aided by the availability of electricity, complemented the skills of the, increasingly literate, manufacturing labour force. A host of new technologies also became available to substitute for labour in the home. Electrical appliances, heating and plumbing decreased the amount of work required for day-to-day living. While innovation in production technologies increased the demand for skilled workers, innovation in home production decreased the labour required in the home (Greenwood, Seshadri and Yorukoglu, 2001).

Early growth in the labour force participation of female workers is related to three, critical, changes in the economy; increased skill level of the workforce, increased division of labour in production, and rapid innovation and adoption of new technologies. Prior to the turn of the century, the small fraction of the workforce with sufficient skills made hiring skilled clerical workers unprofitable. Female workers could only choose, if at all, between employment in manual type waged labour or home production.[6] With high costs of home production, in terms of labour input, relative to wages paid to women in manual labour, few women chose to work in waged employment. As high school attendance and graduation rates increased for both men and women, a reserve of skilled *potential* workers - educated women who were employed in home production - was created. With skill levels of the workforce increasing, firms that could now profitably change their mode of organization to incorporate skilled workers, chose to do so. The increase in the supply of skilled workers also incited increased adoption of technologies utilized by these workers increasing their overall

[6]Where home production includes unpaid agricultural production.

productivity, thereby reducing the costs of production. Increases in the potential supply of skilled workers *induced* both a change in the organization of production and a redirection of investment towards office technologies. That is, initial increases in the supply of skilled workers (a shift in the supply curve) induced an increase in the demand for those same workers (a shift in the demand curve). New jobs, previously unavailable, were created and filled primarily by women who now found it profitable to choose waged employment over home production. Increases in the productivity of clerical workers prevented the wages of the these workers from falling, as their supply grew, and clerical employment continued to expand over time.

The model is developed as follows. A competitive sector produces final output using two, complementary, inputs: manufactured goods and administrative services. Manufactured goods and administration are in turn produced using a variety of intermediates. Suppliers of the intermediates into the production of Goods and Administration earn monopoly profits in the first period in which they produce. Each intermediate into the production of Goods is produced by combining an individual technology with labour. Each intermediate used in the production of Administration is produced using one of two modes of production. If the first mode of organization is used the supplier of the intermediate uses the technology as its only input. If the second mode of organization is used the supplier of intermediate combines an individual technology with skilled labour (clerical workers) to whom the supplier pays a wage. The rate at which suppliers adopt the second mode of organization is a function of the relative profitability of the two modes. Hiring a small number of these workers reduces the marginal product of the entrepreneur, due to costs of supervision for example, and increases costs. When the supply of skilled workers is low, profits to choosing the second mode of organization are low. As the supply increases, however, returns to the division of labour in the production of administrative services makes choosing this mode of organization profitable. Likewise, the rate of return to investing in intermediates in the administrative sector, relative to manufactured goods sector, determines the relative rate at which new technologies are adopted. If the first mode of organization is used, increases in the skill level of the population (both male and female) decrease the profitability of investing in the intermediates in administration relative to manufactured goods. If the hiring of skilled workers becomes profitable, however, increases in both the proportion of suppliers using the new organizational mode and the supply of skilled female workers provides incentives for the adoption of technologies that complement those workers. Wages of skilled female workers, that would have otherwise fallen as their supply increased, rise as households increase the labour they supply to the market.

The introduction of a rising skill level in a model in which economic rents determine both organizational structure and the relative rates of technical adoption leads to complex transitional dynamics in the growth of the economy. For example, human capital growth influences the rate of technical adoption in the sector in which it is

employed by influencing the profitability of investing in that sector. However, it also influences the rate of technical adoption in the sector in which it is not employed both directly, through the redirection of investments away from that sector, and indirectly, through the complementarity of inputs. In addition, both human capital growth *and* the relative rate of technical adoption influence the profitability of hiring clerical workers in administration and, as such, the rate at which suppliers adopt the new organizational structure. Finally the choice of organizational structure feeds back into the relative profitability of investing in intermediates in Administration and influences the relative rate of technical adoption between sectors. All of these factors determine both relative and absolute levels of the wage in the administrative sector and determine the rate at which labour demand increases over time.

To understand these dynamics, the evolution of the economy is examined over four phases. In the first phase, no supplier uses clerical workers in production. Growth in the economy in this phase, if any, stems from increases in effective labour employed in the goods sector. In the second phase, some, but not all, suppliers find it profitable to hire clerical workers but are restricted by the reservation wage required to encourage potential workers to enter the workforce. In phase three, the wage is above the reservation wage, some, but not all, suppliers hire clerical workers but are restricted by the supply of skilled female workers. In phases two and three, growth in the economy, if any, results from both increases in skill level of the population and changes the mode of organization. In phase four, all suppliers use clerical workers in production, all growth is result of increases in skill level of the population.

Figure 1 illustrates a relationship between high school graduation rates and female labour force participation for the nine census divisions of the United States for the period 1910 to 1940. The increase in high school graduation rates was not consistent through-out the country. The change in the proportion of high school graduates is the increase in the percentage of 17 year olds graduating from high school between 1910 and 1940 (from Goldin, 1994). The change in the proportion of females over the workforce is the change in the percentage of workers who were women between 1910 and 1940 (from *Historical Statistics*). As the figure illustrates, the regions of the country with the highest increases in the level of high school graduation rates (i.e. Pacific and Mountain) were also the regions with the highest increase in the proportion of the workforce that are female. Other regions, predominantly in the south, experienced low increases in high school graduate rates and negative growth in female labour force participation.

Figure 2 illustrates the increase in the clerical work force over the period 1870 to 1970.[7] What is notable is not only the increase in the number of workers who could

[7]Sources: Edwards (1943) page 100, U.S Census 1950 and U.S. Census 1970. Note that pre-1940 figures are for the workforce greater than age 10, and for post-1940 are for the workforce greater

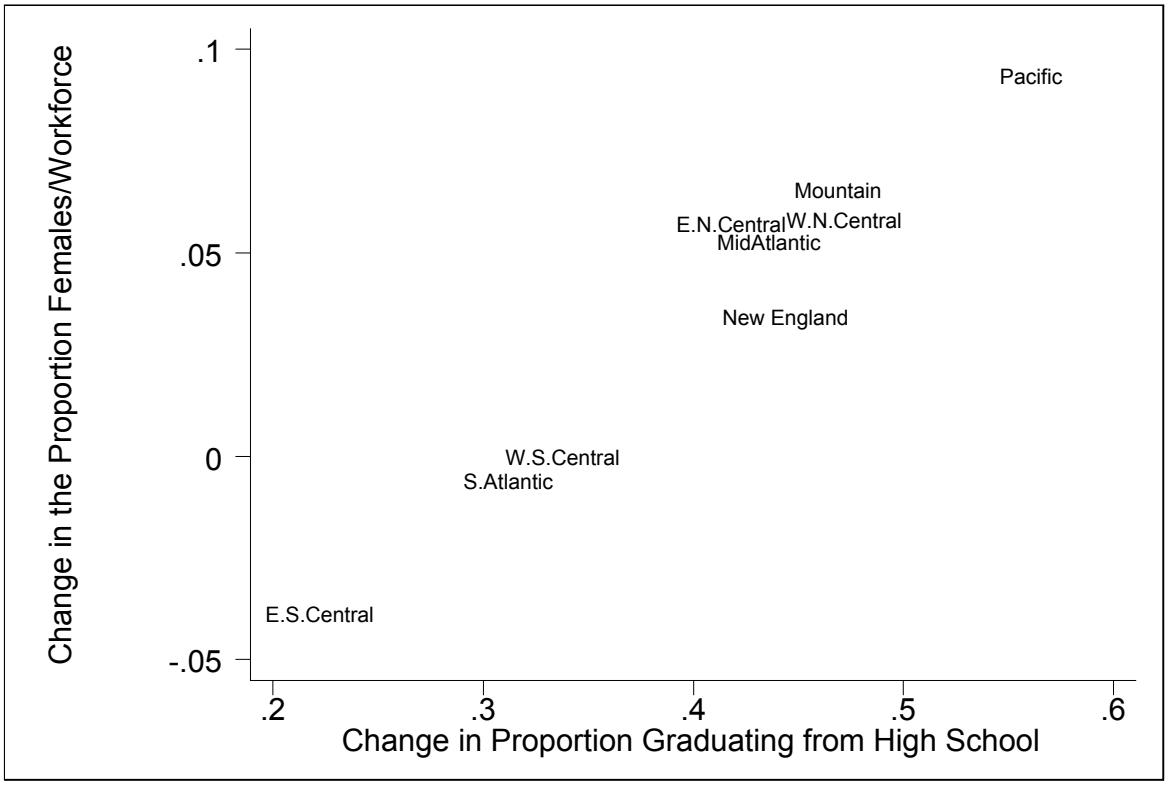

Figure 1: Changes in high school graduation rates and the female labour force over nine census regions between 1910 and 1940.

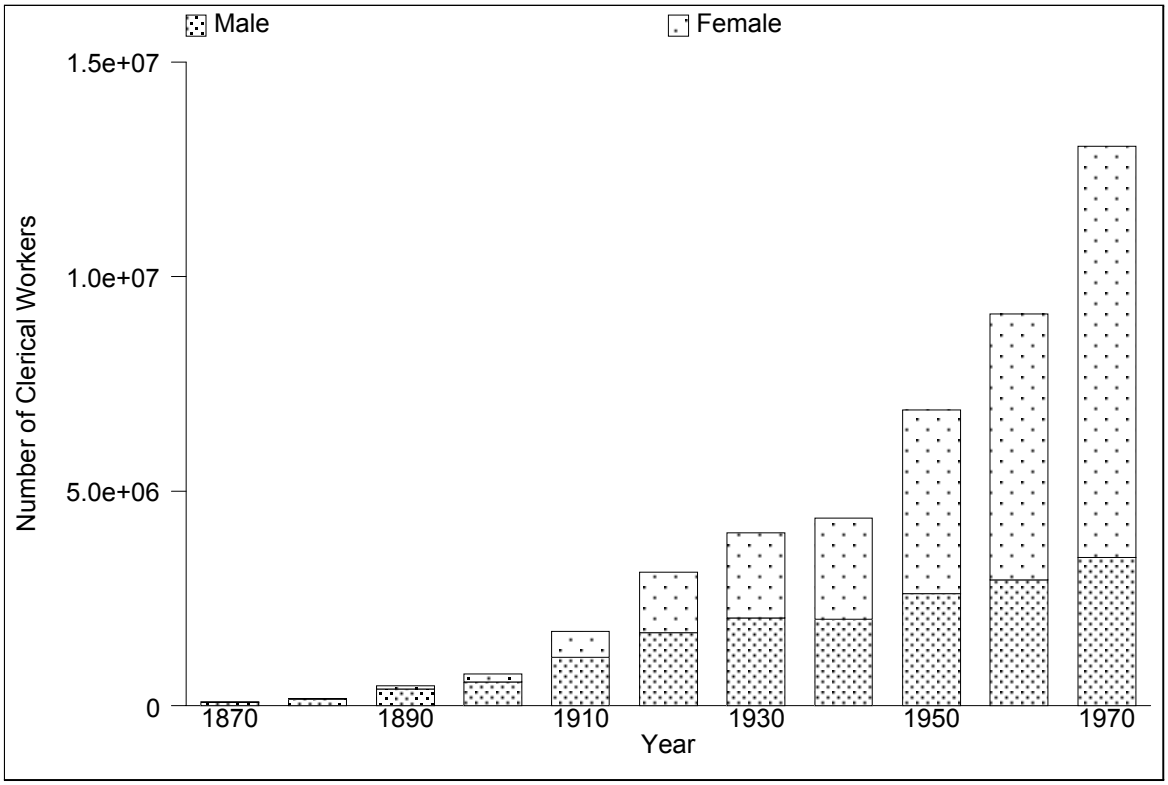

Figure 2: The rise of the clerical workforce, 1870 to 1970.

be classified as clerical workers, but that growth in this sector is predominantly due to an increase in female clerical workers. For more on the increase of female clerical workers see Rotella (1981).

The outline of this paper is as follows. The following section contains a review of the related literature in this area and places this paper within that context. The third section outlines the basic model. The fourth section gives a definition of the equilibrium and the fifth section characterizes it. The sixth section demonstrates how female demand evolves through each phase of the economy and details the transitional dynamics. The seventh section reports the results of a basic simulation of the model and the final section concludes the paper. A list of variables and their definitions follows the conclusion and all proofs are given in the appendix.

2 Related Literature

Claudia Goldin has made contributions to our understanding of female labour force participation not only through "Understanding the Gender Wage Gap" (1990) but through papers (with Lawrence Katz, 1999) in the area of technical change and the returns to education. In "Understanding the Wage Gap" Goldin looks at female labour force participation rates in the US over three periods. Using parameter estimates and relative changes in wages and employment she determines the relative supply and demand effects that lead to greater female labour force participation. In the period 1890 - 1930, Goldin determines that the movement into the workplace was almost exclusively caused by a supply side effect. During 1940 - 1960, the effect is reversed and the demand side effect outweighed the supply side effect, which continued to grow over the period. The rapid growth of sectors that employed women was primarily responsible for this increase in demand, as well as the impact of the second world war. In the final period, 1960 - 1980, increases in both demand and supply caused rapid growth in female participation in the workforce, with the elasticity of supply increasing over time as women's labour supply became more responsive to small changes in the wage.

While "Understanding the Gender Wage Gap" is an extensive empirical analysis of the movement of women into the workforce, the partial equilibrium approach fails to capture the impact of increases in the supply of skilled workers on technological adoption, and ultimately on the demand for those same workers. Acemoglu (1998) and Beaudry and Green (1998) endogenize both labour supply and labour demand in order to examine the effect of market size on "directed technical change". In a

than age 14. The figures pre-1940 were corrected by Edwards for consistency across years. 1940 was adjusted for consistency with 1950 by the U.S. Census Bureau and 1960 was adjusted for consistency with 1970. No other adjustments were made.

general equilibrium framework, increases in the supply of workers with a particular set of skills encourages firms to adopt technologies which complement those workers. This adoption of technologies, in turn, increases the demand for workers who have the skills necessary to utilize that technology.

I extend the work done by Acemoglu and Beaudry and Green to include a change in the mode of organization and to explicitly study the transitional dynamics. By modeling female labour force participation in a general equilibrium setting, and endogenizing both the labour supply and choice of technology, I develop a more precise framework to examine movements into the workforce than found in Goldin's analysis.

Greenwood, Seshadri and Yorukoglu (*Engines of Liberation*, working paper) model women's labour force decisions using a Beckerian model of household production. They argue that women's labour force participation at the turn of the century can be explained by increases in the supply of household durable goods and innovation in the sector that provided substitutes to home production, concluding that labour force participation rates rose steadily over the last century as prices of these goods declined. Innovation in home production arrives exogenously and changes in the prices of durable goods are determined outside of the model. This paper fixes the price of the home production technology to that of the aggregate consumption good to determine if the model can predict the increase in the household labour supply without introducing a market for these goods. Future work will include a model that endogenizes the adoption of home production technologies under the premise that producers of substitutes to home production are responding to a market size effect; as education rates increase and firms adjust their modes of production to accommodate the rise in skilled workers, the demand for these goods increases. In order to fully understand the role of home production technologies, it is first necessary to understand the role of this initial increase in the labour supply in determining the demand for these goods.

3 The Basic Model

The basic model consists of a large number of households making decisions within an overlapping generations framework and a competitive sector producing an aggregate final good using multiple, differentiated, intermediates. While the overlapping generation framework obscures much of the complexity of the household's choices over labour and leisure, and, savings and consumption, by simplifying the household decision we can concentrate on the demand side effects, which are the main focus of this paper.

3.1 The Household

Each household consists of two members, one male and one female.[8] The number of households in each generation is normalized to one and individual households have zero measure. Households live for two periods with preferences over consumption of market produced goods and home produced goods:

$$U(h, c_t, c_{t+1}) = \begin{cases} \overline{h} + \ln(c_t) + \beta \ln c_{t+1} & \text{if } h \geq \overline{h} \\ -\infty & \text{if } h < \overline{h} \end{cases}, \tag{1}$$

where c_t is consumption of a market produced good Y, in period t, β is the discount factor and h is the consumption of household goods and services.

Each individual in the household is endowed with one unit of labour. Individuals allocate their labour between household production and market labour. Time spent working is non-divisible; labour is supplied either to the market or the home but not to both. The production of $\overline{h}$ units of the household good requires the input of exactly one unit of labour when produced at home. If both units of labour are supplied to market the household good must be purchased on the market. Since final goods are the numeraire, the price of the household good equals unity.[9]

The total wage income of the household in period t, I_t, is a function of the total labour sold on the market, and s_t is the savings of the household in period t. When only one member of the household supplies labour to the market, the budget constraint is:

$$c_t + s_t = I_t, \tag{2}$$

and when both supply their labour:

$$c_t + s_t = I_t - \overline{h}. \tag{3}$$

In either case the household supplies no labour to the market in the second period and the budget constraint is:

$$c_{t+1} = (1 + r_{t+1}) s_t \tag{4}$$

[8] By assuming that the household is made of only two individuals - one male and one female - I am assuming the simplest possible household structure. An alternative way to consider this framework is to assume there are two types of workers, one of whom has a comparative advantage in the production of manufactured goods. Using the male/female is notation is consistent, however, with the 'feminization' of clerical work in this period. It is not to say that all clerical employment was female, it wasn't, but the growth in employment in the clerical sector was largely due to increased employment of women in clerical positions.

[9] This too is a simplifying feature of the model; one that could be generalized to allow for relative price changes between home produced and other consumption goods.

Households allocate consumption over time by maximizing (1) subject to (2) and (4) when one member of the household supplies labour to the labour market, and subject to (3) and (4) when both members supply labour to the labour market. Given log linear preferences this yields consumption in the first period given by:

$$c_t = \begin{cases} \frac{I_t - \overline{h}}{1+\beta} & \text{if both work} \\ \frac{I_t}{1+\beta} & \text{otherwise} \end{cases} \tag{5}$$

and savings given by:

$$s_t = \begin{cases} \frac{\beta}{1+\beta}\left(I_t - \overline{h}\right) & \text{if both work} \\ \frac{\beta}{(1+\beta)} I_t & \text{otherwise} \end{cases} \tag{6}$$

Time subscripts are dropped where there is no ambiguity.

3.2 Structure of the Firm

Consumption goods are produced by a large number of competitive firms each producing identical output. Each individual firm has an integrated organizational structure consisting of three levels of production. Producers in every level of production behave as individual profit maximizing firms. Each level of production is discussed in detail below and a diagram with the organizational structure of the firm is in Figure 3.

3.2.1 Final Good Production:

Final good, Y, is produced using constant elasticity of substitution (CES) production technology with intermediate inputs "Goods" (G) and "Administration" (A). The production function for firm i is given by:

$$Y_i = \left[\nu G_i^{\rho} + (1-\nu) A_i^{\rho}\right]^{1/\rho}, \tag{7}$$

where the distributive parameter ν determines the relative importance of each factor in production and ρ is the substitution parameter. The constant elasticity of substitution *between* Goods and Administration is $\sigma = \frac{1}{1-\rho}$. It is natural to assume that Goods and Administration are gross complements in the production of Y, that is $\sigma < 1$ or $\rho < 0$.[10] As firms are competitive and symmetric with constant returns to scale, the subscript i can be dropped.

[10] This assumption is fitting given the context and the specific nature of the two sectors considered. None of the main results of the paper depend on it however. The main results of the paper, with this assumption dropped, are considered in the appendix.

3.2.2 Goods and Administration Production:

Goods and Administration are in turn produced using multiple, differentiated, intermediates. Intermediates are specific to the sector in which they are used and represent the output of individual technologies that are adopted at the beginning of the period. The output of these technologies enter the production functions of G and A in such a way that there are increasing returns to scale in the level of intermediates. This scale effect arises not from increasing returns in the production of intermediates themselves, where production is strictly linear, but from returns to increased differentiation of intermediates. As the variety of intermediates increases, total factor productivity of all inputs increases; as tasks become more specialized (and intermediates more numerous) output of the Goods and Administration increases proportionally faster than the increase in resources allocated to production.[11]

Both Goods and Administration produce output using the Dixit and Stiglitz (1977) production technology:

$$A = \left[\int_0^{M_A} x_A(j)^{\alpha}\, dj\right]^{1/\alpha} \text{ and } G = \left[\int_0^{M_G} x_G(j)^{\alpha}\, dj\right]^{1/\alpha}, \tag{8}$$

where the measure of intermediates are M_A and M_G and the output of intermediate j, in sectors A and G is $x_A(j)$ and $x_G(j)$. The coefficient α determines the level of benefit derived from increased specialization in production. A high α indicates that intermediates are easily substituted within the production of the Goods and Administration. The assumption that this coefficient is identical across sectors is made for simplicity and is dropped in the generalized version of the model.

The variety of intermediates indicates the state of technology in the production of Goods or Administration. If the adoption of new technologies were costless, producers of Goods or Administrative services would demand an infinite variety of intermediates. The creation of intermediates are not costless, as we will see, and, as a result, the variety of intermediates, and the state of technology, is endogenously determined.

The producers of Goods sell their output, G, to producers of the final good at price q_G, producers of Administration sell their final output, A, to producers of the final good at price q_A, and both producers earn zero profits.

3.2.3 Intermediates Production:

The intermediates used in the production of G and A are the output of individual technologies. The intermediates to Goods represent manufactured output that are

[11] For more information on expanding product variety models see Grossman and Helpman (1991) and Romer (1990).

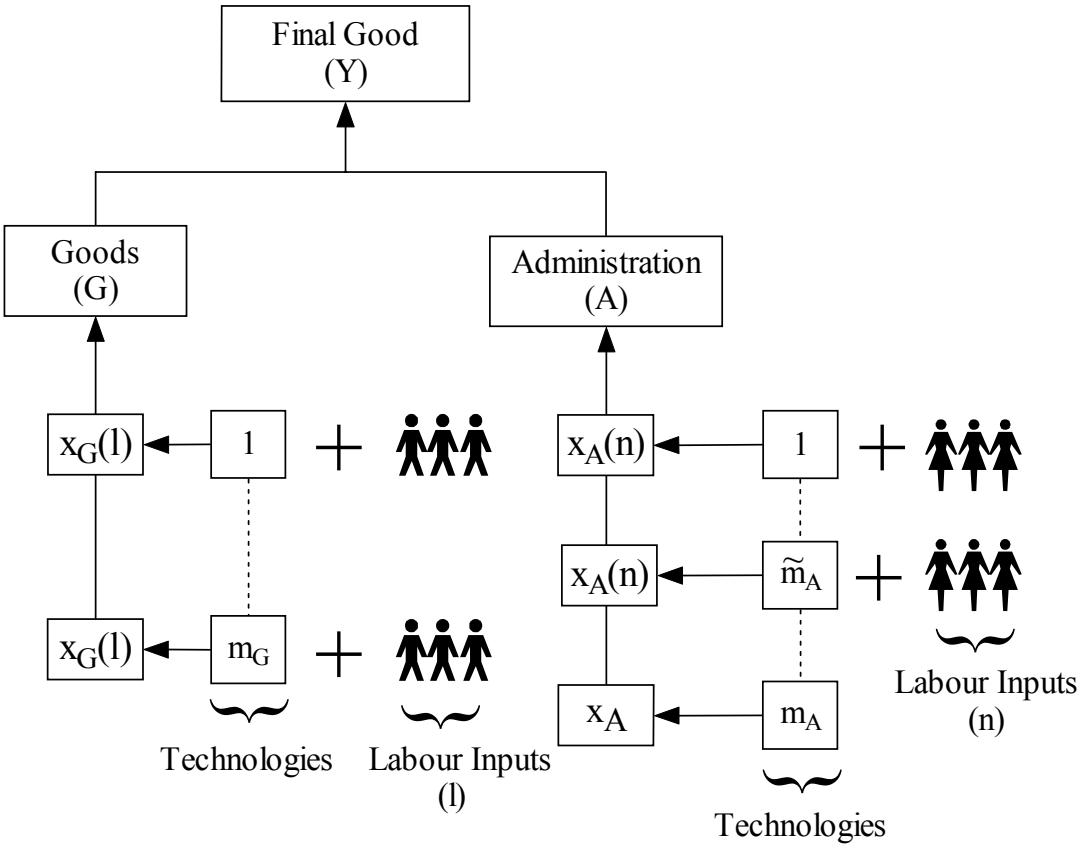

Figure 3: Organizational Structure of the Firm

produced through physical effort, or *brawn* type jobs. The intermediates to Administration are non-manufactured or service-type output that is produced through skill, or *brain* type jobs. Each *technology* exists prior to the beginning of the period in which it is utilized and, as such, suppliers of these intermediates are technology 'adopters' rather than 'innovators'.[12] Each intermediate requires a fixed investment and earns monopoly profits in the first period in which it is produced. In the basic model, technology is adopted for one period, after which it becomes obsolete and is replaced by new technologies. This assumption is dropped in the generalized version of the model where technology becomes obsolete overtime and where intermediates are subject to competition in the second period following their adoption.

Each of the M_G intermediates used in the production of Goods is produced by combining its unique technology with labour. The supplier of intermediate j pays wage $w_G(j)$ per *efficiency unit* of labour input and sells their output to the producer of G for $p_G(j)$. The level of output when labour is combined with technology j is:

$$x_G(j) = \theta l(j), \tag{9}$$

where $l(j)$ represents the efficiency units of labour input to intermediate j in the production of G and θ is the parameter on labour.

As all intermediates are produced with identical production functions and as the wage is set at the competitive level, the index j can be unambiguously dropped so that $w_G(j) = w_G$, $l(j) = l$ and output in sector G is :

$$G = \theta l M_G^{\frac{1}{\alpha}}. \tag{10}$$

Each of the M_A intermediates used in the production of Administration can be produced using one of two modes of production: an 'old' mode of organization or a 'new' mode of organization. The old mode of organization uses no labour input. Output with this mode of organization is:

$$x_A(j) = \theta. \tag{11}$$

The new organizational mode uses the labour of skilled workers. The second mode of organization employed is:

$$x_A(j) = \theta n(j). \tag{12}$$

where $n(j)$ is the efficiency unit of labour input when when technology j is used.

[12] In the sense that it is the rate of adoption of new technologies in each sector that determines the outcome of the model, rather than innovations. The terms 'technology' and 'capital' are interchangeable here.

As in the production of intermediates to G, all intermediates are produced with identical production functions, the wage is set at the competitive level and the index j can be unambiguously dropped so that $w_A(j) = w_A$ and $n(j) = n$.[13]

In every period, with M_A suppliers of intermediates to sector A, there are $\widetilde{M}_A$ suppliers using the new organizational mode and $M_A - \widetilde{M}_A$ suppliers using the old mode of organization. Output in sector A is:

$$A = \theta \left[\widetilde{M}_A n^{\alpha} + \left(M_A - \widetilde{M}_A \right) \right]^{\frac{1}{\alpha}}. \tag{13}$$

Clearly, if $\widetilde{M}_{A,t} = 0$, then $A = \theta M_A^{\frac{1}{\alpha}}$.

3.3 Human Capital

Individuals in this economy can be either skilled or unskilled.[14] The effective aggregate human capital supplied by male workers given the stock of unskilled male workers, L_u^m, and skilled male workers, L_s^m, is:

$$H^m = a_u L_u^m + a_s L_s^m, \tag{14}$$

where a_u denotes efficiency units of unskilled male labour and a_s efficiency units of skilled male labour.

The *potential* human capital supplied by female workers (i.e. if all women choose to work in the waged workforce) given the stock of unskilled female workers, L_u^w, and skilled female workers, L_s^w, is:

$$H^w = b_u L_u^w + b_s L_s^w, \tag{15}$$

where b_u denotes efficiency units of unskilled female labour and b_s efficiency units of skilled female labour.

Skilled workers are more productive ($a_s > a_u$ and $b_s > b_u$) and male workers have a comparative advantage in employment in the production of the intermediate to G. Where $\left\{a_u^G, a_s^G, b_u^G, b_s^G\right\}$ are the productivity parameters on labour in the Goods sector and where $\left\{a_u^A, a_s^A, b_u^A, b_s^A\right\}$ are the productivity parameters on the labour in the Administration sector, then

$$\frac{a_s^G}{a_s^A} > \frac{b_s^G}{b_s^A} \tag{16}$$

[13] It is not necessary for the productivity parameter, θ, be the same for both modes of production. This assumption is dropped in the generalized version of the model.

[14] Where workers are skilled in the sense that they possess the basic level of literacy and numeracy skills attributed to individuals who have completed, or nearly completed, a high school education.

imposes this condition. To simplify the analysis I set $a_u^A = b_u^A = 0$, so that employment in the production of the intermediates to A is restricted to sufficiently skilled workers. Note that no restrictions have been placed on the absolute advantage of workers.

Given the wages available to workers (w_A and w_G) and the productivity parameters defined above, for households to be indifferent between supplying one more unit of labour to the market and supplying one more unit of labour to the home it must be the case that the wage paid for that labour input must equal $\overline{h}$. It is also the case that as long as $b_u^G w_G < \overline{h}$ and $a_u^G > b_u^G$, unskilled female workers will supply their labour to home production. The potential efficient female labour supply, in period t, is:

$$H^w = \begin{cases} 0 & \overline{h} > b_s^A w_A, \overline{h} > b_s^G w_G \\ b_s^A L_s^w & b_s^G w_G < \overline{h} < b_s^A w_A \\ b_u^G L_u^w + b_s^A L_s^w & \overline{h} < b_u^G w_G, \overline{h} < b_s^A w_A \end{cases} . \tag{17}$$

Let the wage at which skilled female workers are indifferent between supplying labour to the household and supplying skilled labour to the market be $\overline{w}$;

$$\overline{w} = \frac{\overline{h}}{b_S^A}. \tag{18}$$

4 Definition of Equilibrium

An equilibrium is determined in every period by the following conditions. The first section determines the conditions under which individual suppliers are behaving optimally. The second section determines the allocation of investment between the Goods and Administration sectors when the capital market is clearing, and when households take the measure of suppliers using the new mode of organization as given. The third section determines the optimal decision of suppliers of intermediates to sector A on the mode of organizational choice for a given level of available human capital and the reservation wage when suppliers take the level of investment in the sector as given. Conditions required for the division of occupation by gender are determined in the fourth section.

4.1 General Conditions

- The producer of the final good chooses inputs A and G so as to minimize costs. Free entry then implies that derived demand for intermediates A and G is:

$$A = Y\left(\frac{q_A}{1-\nu}\right)^{\frac{1}{\rho-1}} \text{ and } G = Y\left(\frac{q_G}{\nu}\right)^{\frac{1}{\rho-1}}. \tag{19}$$

- Producers of Goods choose intermediates x_G so as to minimize costs. Free entry then implies that derived demand for intermediate x_G is:

$$x_G = \left(\frac{q_G}{p_G}\right)^{\frac{1}{1-\alpha}} G. \tag{20}$$

- Suppliers of the intermediates to G choose the price that maximizes profits given the level of derived demand above. The monopoly price and output are:

$$p_G = \frac{w_G}{\theta\alpha} \tag{21}$$

and

$$x_G = \left(\frac{\alpha\theta}{w_G} q_G\right)^{\frac{1}{1-\alpha}} G \quad (= \theta l). \tag{22}$$

- The wage paid to workers in the Goods sector is:

$$w_G = \frac{\alpha\theta^{\alpha}}{l^{1-\alpha}} \nu Y^{1-\rho} G^{\rho-\alpha}. \tag{23}$$

- Producers of A choose intermediates, x_A, so as to minimize costs. Free entry implies that derived demand for intermediate x_A is:

$$x_A = \left(\frac{q_A}{p_A}\right)^{\frac{1}{1-\alpha}} A. \tag{24}$$

- Suppliers of the intermediates to A who use the new organizational mode choose the price which maximizes profits given the level of derived demand above. The monopoly price and output are:

$$p_A = \frac{w_A}{\theta\alpha} \tag{25}$$

and

$$x_A = \left(\frac{\theta\alpha}{w_A} q_A\right)^{\frac{1}{1-\alpha}} A \quad (= \theta n). \tag{26}$$

- The wage paid to workers in sector A is:

$$w_A = \frac{\theta^{\alpha}\alpha}{n^{1-\alpha}} (1-\nu) Y^{1-\rho} A^{\rho-\alpha}. \tag{27}$$

4.2 Conditions on The Relative Rate of Technology Adoption

In each period, the relative rate of technology adoption in the Goods and Administration sectors is determined by equating profits across sectors and by equating investment in new technologies to savings from the previous period. Households make investment decisions in every period taking the measure of suppliers to A using the new mode of organization as given.

- Capital market clearing implies that investment in every period is equal to savings from the previous period. Given γ is the cost of a technology in both sectors and $M_{G,t+1}$ and $M_{A,t+1}$ are the measure of intermediates in period $t+1$, total investment equals *total* savings (S_t) in period t when:

$$\gamma M_{G,t+1} + \gamma M_{A,t+1} = S_t. \tag{28}$$

- No-arbitrage conditions are such that the return to investment must be equal across sectors. The relative rate of technology adoption in the Goods and Administration sectors is determined by equating profits across sectors. This condition is simply:

$$\frac{\pi_{G,t}}{\gamma} = \frac{\pi_{A,t}}{\gamma} = 1 + r_{t-1}. \tag{29}$$

- The return to investing in technology M_G is:

$$\pi_G = \nu \left(1-\alpha\right) \left(\theta l\right)^{\alpha} Y^{1-\rho} G^{\rho-\alpha}. \tag{30}$$

- The return to investing in technology M_A that uses the old mode of organization is:

$$\pi_A^O = \left(1-\nu\right) \theta^{\alpha} Y^{1-\rho} A^{\rho-\alpha}. \tag{31}$$

- The return to investing in technology M_A that uses the new organizational mode is:

$$\pi_A^N = \left(1-\nu\right) \left(1-\alpha\right) \left(n\theta\right)^{\alpha} Y^{1-\rho} A^{\rho-\alpha}. \tag{32}$$

Note that when $\widetilde{M}_A < M_A$ it must be the case that $\pi_A^N \leq \pi_A^O$, profits in the new mode are at most equal to profits in the old mode for a given level of human capital. If the reverse were true no supplier would use the old mode of organization. When $\widetilde{M}_A = M_A$, and all suppliers find it profitable to use the new organizational mode, then $\pi_A^N \geq \pi_A^O$. The no-arbitrage condition in (29) therefore holds when $\pi_A^O = \pi_G$ if $\widetilde{M}_A \leq M_A$ and $\pi_A^N = \pi_G$ when $\widetilde{M}_A = M_A$. Given the anticipated measure of suppliers using the new organizational mode, the conditions on the relative rate of technology adoption determine the level of investment in a sector depending on whether or not $\widetilde{M}_A = M_A$.

4.3 Conditions on the Choice of Organizational Mode of Production

Suppliers choose the organizational mode which returns the greatest profits in that period, taking the level of technology in sector A as given. The profitability of the new mode of organization is determined by both the level of potential female human capital and the market wage. If the potential demand is such that the market wage falls below the reservation wage then this mode is not feasible. If the potential demand is such that the market wage exceeds the reservation wage, a supplier will still only use the new organizational mode if the available supply of available human capital is sufficiently high to make production sufficiently profitable.

- Solving (26) for n, setting the wage equal to $\overline{w}$ and substituting into (32), a single deviating supplier choosing the new organizational mode earns profits:

$$\pi_A^N(\overline{w}) = (1-\nu)(1-\alpha)\left(\left(\frac{\alpha\theta}{\overline{w}}\right)^{\alpha} Y_t^{1-\rho} A_t^{\rho-\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{33}$$

- The existence of an equilibrium in which at least one supplier deviates to the new organizational mode requires that $\pi_A^O < \pi_A^N(\overline{w})$. Using (31) and (33) this condition can be expressed as :

$$\overline{w} < \alpha\theta^{\alpha}\left((1-\nu)(1-\alpha)\right)^{\frac{1-\alpha}{\alpha}} Y^{1-\rho} A^{\rho-\alpha}. \tag{34}$$

  If (34) does not hold, an equilibrium exists in which no suppliers choose the new organizational mode.

- If demand is such that the market wage meets or exceeds the reservation wage, the condition on the division of labour in the production of intermediates to A is met when:

$$\pi_A^N \geq \pi_A^O. \tag{35}$$

4.4 Condition on Male Labour Not Entering the Administration Sector

The necessary condition required when male workers do not enter the clerical workforce is given by:

$$\frac{a_s^G}{a_s^A} w_G > w_A > \overline{w}. \tag{36}$$

Given w_A in (27) and $w_{G,t}$ in (23) this condition is simply:

$$\frac{a_s^G}{a_s^A} > (1-\alpha)^{\frac{1}{\alpha}} \frac{M_{G,t}}{H_t^m}. \tag{37}$$

5 Characterization of Equilibrium

As we have seen, the relative rate of technology adoption in Goods and Administration is determined by equating profits across sectors and equating investment in new technologies to savings from the previous period. The measure of suppliers using the new mode of organization, on the other hand, is a function of the market wage, the reservation wage, and the availability of skilled workers. In equilibrium, households choose the optimal allocation of investment across sectors conditional on the *anticipated* measure of suppliers using the new mode of organization. Subsequently suppliers of intermediates choose the optimal mode of organization conditional on these investments.

In order to provide a stationary representation of the equilibrium it convenient to deflate all variables by male human capital H^m. Let:

$$m_A = \frac{M_A}{H^m}, \tag{38}$$

$$\widetilde{m}_A = \frac{\widetilde{M}_A}{H^m}, \tag{39}$$

$$s = \frac{S}{H^m}, \tag{40}$$

$$y = \frac{Y}{H^m}, \tag{41}$$

$$a = \frac{A}{H^m}. \tag{42}$$

Let the ratio of technology in sector G to male human capital be denoted by:

$$\chi = \frac{M_G}{H^m}, \tag{43}$$

and let male human capital grow at rate g^m such that:

$$H_t^m = (1+g^m)\, H_{t-1}^m. \tag{44}$$

5.1 The RTA Curve

How m_A depends on the anticipated level of $\widetilde{m}_A$ depends on whether or not $\widetilde{m}_A \leq m_A$. If $\widetilde{m}_A < m_A$, the returns to investing in the Goods sector are given by (30) and the returns to investing in the Administrative sector are given by (31). The no-arbitrage condition in (29), therefore, holds if:

$$A_t = \left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{1}{\alpha-\rho}} (\chi_t)^{\frac{\alpha}{\alpha-\rho}} G_t, \tag{45}$$

where $l_t = \frac{H^m}{M_G} \left(= \frac{1}{\chi_t}\right)$; all male human capital is employed by M_G suppliers of intermediates.

Substituting deflated values into (28) and rearranging:

$$\chi_t = \frac{1}{\gamma} s_{t-1} - m_{A,t}. \tag{46}$$

Where output in sector G is given by (10) is:

$$G_t = \theta H_t^m \left(\frac{1}{\gamma} s_{t-1} - m_{A,t}\right)^{\frac{1-\alpha}{\alpha}}, \tag{47}$$

substituting (47) and (13) into (45) yields the measure of technology in Administration as a function of savings and the measure of suppliers using the new mode, when $\widetilde{m}_{A,t} < m_{A,t}$, this is given by $m_A = m_A\left(\widetilde{m}_{A,t}, s_{t-1}\right)$ implicitly defined by:

$$m_A = \left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{\alpha}{\alpha-\rho}} \left(\frac{1}{\gamma} s_{t-1} - m_A\right)^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} - \widetilde{m}_{A,t}\left(n_t^{\alpha} - 1\right). \tag{RTA}$$

The equation in (RTA) maps out a relationship between m_A and $\widetilde{m}_A$ (for a given level of savings) at every point at which profits are equal across sectors. Plotting this line in $m_A - \widetilde{m}_A$ space yields a curve that is both downward sloping (for all $n > 1$) and shifting to the right as savings increases. If the level of output per technology was equal across modes of production (as is the case when $n = 1$), then a fixed proportion of savings would be invested in sector A technology. If $n > 1$ then suppliers of the intermediates to A using the new organizational mode produce more output, per technology then suppliers using the old mode. Everything else held constant, a marginal increase in $\widetilde{m}_A$ increases the average output per technology in sector A, putting downward pressure on profits in that sector and reducing the overall level of investment. An increase in savings, however, increases the level of investment in sector A for any given measure of suppliers using the new mode of organization. Where this curve determines the relative rate of technology adoption, in Figure 4 this

curve is labeled RTA. For a given level of savings, RTA is negatively sloped at every point below the 45° line[15] and shifts up and to the right when saving increases.

If $\widetilde{m}_A = m_A$, the returns to investing in the Goods sector are given by (30) and the returns to investing in the Administrative sector are given by (32). The no-arbitrage condition in (29), therefore, holds when:

$$A_t = \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\alpha-\rho}} \left(\frac{H_t^w}{H_t^m}\right)^{\frac{\alpha}{\alpha-\rho+\alpha\rho}} G_t, \tag{48}$$

where $n = \frac{H^w}{M_A}$; all female human capital is employed by M_A suppliers of intermediates.

If output in G is determined by (47) and output in A is determined by (13), when the profit condition is (48) the level of investment in A as a function of savings satisfies:

$$m_A\left(H_t^w, H_t^m, s_{t-1}\right) = \frac{\left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\alpha-\rho}}}{\left(\left(\frac{H_t^m}{H_t^w}\right)^{\frac{\alpha\rho}{\alpha-\rho+\alpha\rho}} + \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\alpha-\rho}}\right)} \frac{1}{\gamma} s_{t-1}. \tag{RTA4}$$

If $\widetilde{m}_{A,t} = m_{A,t}$ all suppliers of the intermediates to A are using the new organizational mode. Now the marginal supplier using the new mode has no impact on the level of average output. The level of technology in sector A is proportional to savings and is a function of the relative level of female to male human capital.

5.2 The MRW Curve

How $\widetilde{m}_A$ depends on m_A depends on the level of the reservation wage. Substituting deflated values of Y and A into (34) and solving with equality, the level of m_A at which the wage is exactly $\overline{w}$, is given by $m_A = m_A\left(\overline{w}, \widetilde{m}_A, s_{t-1}, H^m\right)$ implicitly defined by:

$$\left[a\left(m_A, \widetilde{m}_A\right)\right]^{\alpha-\rho} = \frac{\alpha\theta^\alpha}{\overline{w}} \left((1-\nu)(1-\alpha)\right)^{\frac{1-\alpha}{\alpha}} \left(H^m\right)^{1-\alpha} \left[y\left(m_A, \widetilde{m}_A, s\right)\right]^{1-\rho}. \tag{MRW}$$

MRW determines not only the level of demand at which a single deviating supplier switches to the new organizational mode, but also determines at which point along RTA is an equilibrium. Where this implicit function determines the mode of organization when the wage is exactly the reservation wage, in Figure 4 this curve

[15] And is not defined elsewhere.

is labeled MRW and is assumed to be both negatively sloped[16] and shifting to the right when both male human capital and savings increase.

5.3 The MHC Curve

If the market wage meets or exceeds the reservation wage, the condition on the division of labour in the production of A is met when $\pi_A^N \geq \pi_A^O$. Using equations (32) and (31), an individual supplier chooses the new organizational mode if the level of labour supply available to that supplier satisfies:

$$n \geq \widetilde{n} = \left(\frac{1}{1-\alpha}\right)^{\frac{1}{\alpha}} > 1. \tag{49}$$

Profits in the new mode exceed profits in the old mode if, and only if, $n_t > \widetilde{n}$. Arbitrage conditions are such that suppliers choose the new organizational mode until profits are equal across modes. As long as $\widetilde{m}_A < m_A$, all suppliers using the new mode use exactly $n = \widetilde{n}$. If $\widetilde{m}_A = m_A$ then this condition no longer binding at equality.

If $\widetilde{m}_A < m_A$, and if all human capital is employed by the $\widetilde{m}_A$ intermediates using the new organizational mode, then the level of labour input per supplier is:

$$n = \frac{H^w}{\widetilde{M}_A} = \widetilde{n}. \tag{50}$$

It follows that the maximum measure of intermediates using the new organizational mode is simply:

$$\widetilde{m}_A\left(n_t, H_t^w, H_t^m\right) = \begin{cases} (1-\alpha)^{\frac{1}{\alpha}} \frac{H^w}{H^m} & \text{if } \widetilde{m}_A < m_A \\ n_t^{-1} \frac{H_t^w}{H_t^m} & \text{with } n_t > \widetilde{n}, \text{ if } \widetilde{m}_{A,t} = m_{A,t} \end{cases}. \tag{MHC}$$

When $m_A < (1-\alpha)^{\frac{1}{\alpha}} \frac{H^w}{H^m}$, the condition on the division of labour does not hold with equality, $n > \widetilde{n}$, and the available supply of labour is at every point along the 45° line. Where $m_A \geq (1-\alpha)^{\frac{1}{\alpha}} \frac{H^w}{H^m}$ the condition on the division of labour is binding, $n = \widetilde{n}$, and the available supply of labour is exactly $(1-\alpha)^{\frac{1}{\alpha}} \frac{H^w}{H^m}$. Where the equation in (MHC) determines the choice of organizational mode given the available human capital, on Figure 4 this curve is labelled MHC.

[16] This is a reasonable assumption to make given that an increase in m_A increases $a\left(m_A, \widetilde{m}_A\right)$ and will either increase or decrease $y\left(m_A, \widetilde{m}_A, s_{t-1}\right)$. If the net effect of an increase in m_A is an increase in the right-hand side of (34) this implies that in order for the equality to hold there must be a corresponding decrease in $\widetilde{m}_A$.

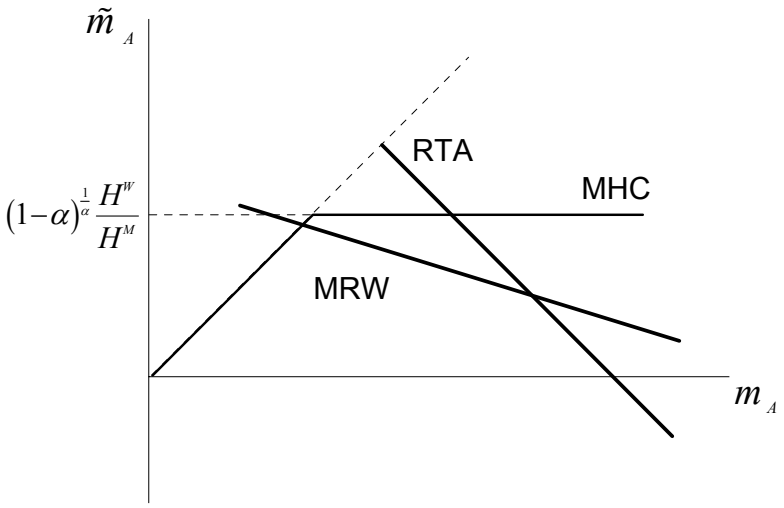

Figure 4: Illustration of an Equilibrium with RTA, MHC and MRW.

5.4 Characterization of the Equilibrium with RTA, MRW and MHC

At every point along RTA the condition on the relative rate of technology adoption is satisfied for a given level of $\widetilde{m}_A$. The line MRW plots every point at which $\widetilde{m}_A$ suppliers are willing to pay exactly the reservation wage, for a given level of m_A. The line MHC is the available relative supply of female human capital. At the point where RTA intersects the x-axis is the level of technology adoption when no suppliers used the new organizational mode and the condition on the relative rate of technology adoption is satisfied. The point where RTA intersects MRW the wage is exactly $\overline{w}$ and the conditions on the relative rate of technology adoption are satisfied. The point where RTA intersects MHC all available female human capital is employed in sector A and the conditions on the relative rate of technology adoption are satisfied. In equilibrium, it must be the case that both the condition on the relative rate of technology adoption is satisfied in every period (i.e. the equilibrium is a point $\{m_A, \widetilde{m}_A\}$ on the line RTA) and the level of human capital employed is less than or equal to MHC. As we will see, the point at which MRW intersects RTA determines not only the level of employment when the equilibrium wage is exactly $\overline{w}$, but also which phase the economy is in when $w_A \neq \overline{w}$.

The wage at every point along RTA is determined by substituting condition (45)

into the wage function (27), and solving for the wage as a function of χ and H^m:

$$w_A(\chi) = \frac{\theta\alpha(1-\alpha)}{n^{1-\alpha}}\nu H^{m\frac{1-\alpha}{\alpha}}\left(\nu + (1-\nu)\left((1-\alpha)^{-1}\left(\frac{1-\nu}{\nu}\right)\chi^{\alpha}\right)^{\frac{\rho}{\alpha-\rho}}\right)^{\frac{1-\rho}{\rho}}\chi^{\frac{1-2\alpha}{\alpha}}. \tag{WAGE}$$

Where $\chi = \frac{1}{\gamma}s_{t-1} - m_{A,t}$.

Lemma 1 is required to determine the conditions under which the demand curve for labour in sector A is downward sloping. Given these conditions it can be shown that there is a unique wage at every point $\{m_A, \widetilde{m}_A\}$ along RTA and that this wage is increasing in m_A and decreasing in $\widetilde{m}_A$.

Lemma 1: *There exists a χ_0, such that if (i) $\chi_t > \chi_0$ and (ii) $\alpha > \frac{1}{2}$, then the clerical wage, w_A, is decreasing in $\widetilde{m}_A$, when $\widetilde{m}_A < m_A$.*

$$\chi_0 = \left(\left(\frac{\alpha}{2\alpha-1}\frac{\alpha(1-\rho)}{\alpha-\rho}\frac{(1-\nu)}{\nu} - 1\right)\left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{\rho}{\alpha-\rho}}\right)^{\frac{\alpha-\rho}{\alpha\rho}} \tag{51}$$

If the conditions in Lemma 1 are satisfied, the wage at every point on RTA is increasing as we move down the curve. The conditions in Lemma 1 are also sufficient to determine that the curve MRW intersects RTA at one point only and that this intersection determines the point at which the wage on RTA is exactly $\overline{w}$. For every level of $\widetilde{m}_A$ on RTA above the point at which RTA intersects MRW, the wage determined by (WAGE) is below the reservation wage. For every level of $\widetilde{m}_A$ on RTA below the point at which RTA intersects MRW, the wage determined by (WAGE) is above the reservation wage. Letting the point at which $RTA = MRW$ be $\{m_A^*, \widetilde{m}_A^*\}$ the equilibrium level of $\widetilde{m}_A$ is determined as follows:

$$\widetilde{m}_{A,t} = \begin{cases} 0 & \widetilde{m}_A^* < 0 \\ \widetilde{m}_A^* & 0 < \widetilde{m}_A^* < \widetilde{m}_A(n, H^w, H^m) \\ n\frac{H^w}{H^m} & \widetilde{m}_A^* > \widetilde{m}_A(n, H^w, H^m) \end{cases}. \tag{52}$$

If MRW intersects RTA below the x-axis, no supplier chooses the new organizational mode and $\widetilde{m}_A = 0$. If MRW intersects RTA above the x-axis, but below MHC, suppliers pay exactly $\overline{w}$ and some available female human capital remains unemployed. If MRW intersects RTA above MHC, the wage when all human capital is employed is above the reservation wage, and the measure of suppliers choosing labour as an input is $\widetilde{m}_A(\widetilde{n}, H^w, H^m)$ if $\widetilde{m}_A < m_A$ and $\widetilde{m}_A(n(>\widetilde{n}), H^w, H^m)$ if $\widetilde{m}_A = m_A$.

Proposition 1 shows that an increase in the supply of male human capital in the production of G is sufficient to induce an increase in proportion of suppliers in A who adopt the new organizational mode.

Proposition 1: *The proportion of suppliers in the Administration sector who would choose to switch to the new organizational mode and pay exactly the reservation wage, $\overline{w}$, is increasing in the level of male human capital given the conditions in Lemma 1.*

Proof: From the condition in (34), in any one period, the ratio of technology in the Goods sector to male human capital when the wage is exactly $\overline{w}$ is the unique $\overline{\chi}_t$ that solves the equality:

$$\frac{\overline{w}}{\theta\alpha\left(1-\alpha\right)^{\frac{1}{\alpha}}H_t^{m\frac{1-\alpha}{\alpha}}} = \left(\nu+(1-\nu)\left(\left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)\left(\overline{\chi}_t\right)^{\alpha}\right)^{\frac{\rho}{\alpha-\rho}}\right)^{\frac{1-\rho}{\rho}}\overline{\chi}_t^{\frac{1-2\alpha}{\alpha}} \tag{53}$$

According to the conditions given in Lemma 1, the term on the right-hand side of the equality is strictly decreasing in χ. An increase in male human capital, which decreases the constant on the left-hand side of the equation, increases the $\overline{\chi}_t$ which solves the equality.

Substituting χ_t in for $\frac{1}{\gamma}s_{t-1}-m_{A,t}$, solving (RTA) for the proportion of suppliers using the new organizational mode as a function of χ, and setting $\chi_t=\overline{\chi}_t$, the ratio of suppliers using the new organizational mode to technology in the sector when the market wage is exactly the reservation wage is:

$$\frac{\widetilde{m}_A}{m_A} = \frac{1}{n_t^{\alpha}-1}\left(\left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{\alpha}{\alpha-\rho}}\left(\overline{\chi}_t\right)^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}}\right), \tag{54}$$

Clearly the proportion of suppliers using the new organizational mode is increasing with $\overline{\chi}_t$ when $\rho<0$.

The proportion of suppliers in sector A who would switch to the new organizational mode and pay exactly $\overline{w}$ is increasing in the level of male human capital. Given that the constraint on the available supply of human capital is met, an increase in male human capital is sufficient to induce a change in the mode of organization.

6 Evolution of Female Labour Demand

Female labour demand evolves over time. As human capital grows the conditions on profitability between the sectors and between organizational modes determine the rate at which suppliers in the administration sector adopt new technologies that utilize skilled labour. Allowing human capital to grow over time, and by computing the equilibrium levels of investment in each sector and the proportion of suppliers using the new organizational mode in every period, the evolution of female labour demand can be traced over time. I consider four phases of the economy over which the level

male and female human capital are growing. In the first phase, the level of human capital is low. The wage at which a single supplier would be willing to switch to the new organizational mode is below the reservation wage. No supplier uses the new organizational mode and no women work. In the second phase, some suppliers find it profitable to switch and pay exactly the reservation wage. The measure of suppliers that adopt the new organizational mode and employ $\widetilde{n}$ labour is below the measure at which all female workers are employed. The level of human capital employed falls below the potential level of human capital. In the third phase, the market wage is bid up above the reservation wage. All suppliers that find it profitable to use the new organizational mode, and pay $w_A > \overline{w}$, do so. The measure of suppliers who switch in this phase is constrained by the supply of skilled workers. In phase four, the level of female human capital is sufficiently high that all suppliers use the new organizational mode, pay $w_A > \overline{w}$ and employ $n > \widetilde{n}$ workers.

In each of the following sections, I examine how these factors change over time and describe the transitional dynamics of the economy.

6.1 Phase One: No-Clerical Employment

In phase one, no technology employs clerical workers. Substituting $\widetilde{m}_{A,t} = 0$ into (RTA), the measure of intermediates in A is proportional to the measure of intermediates in G according to:

$$m_{A,t} = \left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{\alpha}{\alpha-\rho}} (\chi_t)^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}}, \tag{55}$$

and the relative rate of growth in sector A is given by the expression:

$$\Delta \log (m_A) = \frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}\Delta \log \chi. \tag{56}$$

The parametric term on the right hand side of the equation is the point elasticity of substitution between m_A and m_G in the production of final goods in phase one. It is a function of the level of complementarity between the sectors (measured by ρ) and the returns to specialization in individual sectors (measured by α). Where $\alpha > 0$, and if the inputs in the production of the final good are gross complements (i.e. $\rho < 0$), growth in Goods will be accompanied by positive, but proportionally slower, growth in Administration.

Proposition 2: *There exists a $\chi_t^1 > \overline{\chi}_t$, such that if $\chi \geq \chi_t^1$, i) an equilibrium exists in which no suppliers use the new organizational mode, and in that equilibrium*

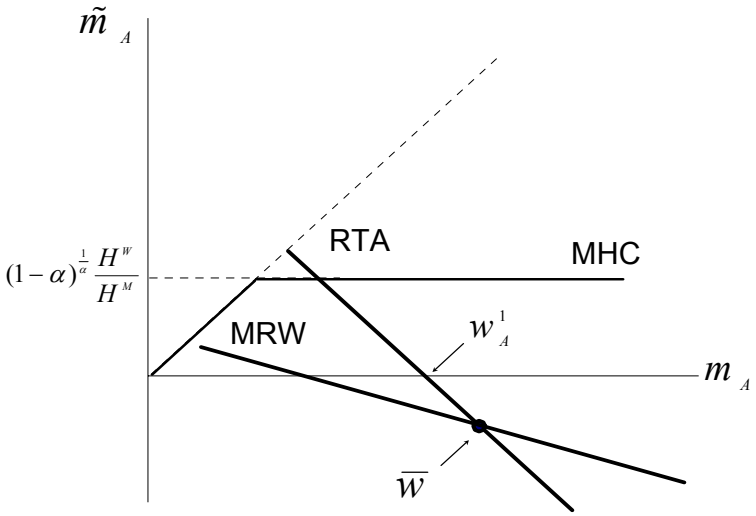

Figure 5: A Phase One Equilibrium

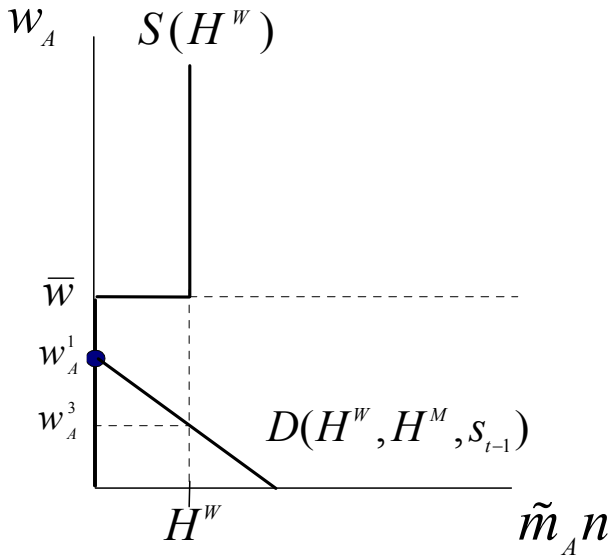

Figure 6: Labour Demand in a Phase One Equilibrium

both ii) the variety of intermediates in G is growing proportionally faster than human capital (χ is increasing over time) and *iii) the growth rate of intermediates in Administration is slow relative to that in Goods.*

A phase one equilibrium is illustrated in Figure 5. In phase one, the point at which RTA intersects MRW falls below the x-axis. At every point along RTA, where $\widetilde{m}_A \geq 0$, the potential demand is such that the market wage is below the reservation wage. The only equilibrium is one in which $\widetilde{m}_A = 0$, no supplier uses the new organizational mode and no female human capital is employed.

As Proposition 2 demonstrates, in phase one, χ is increasing as human capital increases over time. Given the relationship between m_A and χ in (56), and for values of $\alpha > 0$ and $\rho < 0$, the curve RTA shifts up and to the right when human capital increases. Given an increase in savings and human capital, if RTA intersects the x-axis at a point at which $\chi \leq \overline{\chi}_t$ then the economy is no longer in phase one.

Figure 6 illustrates the equilibrium outcome in phase one in a supply/demand framework. The supply of labour to sector A, $S(H^w)$ is equal to zero at all levels of the wage below $\overline{w}$. At wages above $\overline{w}$ the labour supply is perfectly inelastic: all female workers supply the same level of labour regardless of the wage.[17] In phase one, the labour demand curve lies below $\overline{w}$ for every level of employment. The equilibrium outcome is $\widetilde{m}_A n = 0$, no clerical workers are employed.

6.2 Phase Two: Partial Clerical Employment and Surplus Female Human Capital

In phase two, the condition on the reservation wage given in (34) now holds with equality; at least one supplier in A chooses to deviate to the new organizational mode and pay $\overline{w}$. If all suppliers who could profitably choose the new organizational mode, at any wage, did so, however, the wage would fall below the reservation wage and no labour would be supplied. In phase two, some suppliers who could have profitably chosen the new organizational mode at any wage choose to use the old organizational mode and not all available human capital is used in production.

Figure 7 illustrates the equilibrium in phase two. An increase in savings has shifted both RTA and MRW up and to the right. If the increase in human capital is such that $w_A^1 > \overline{w}$, then at least one supplier has deviated to the new organizational mode and $\widetilde{m}_{A,t} > 0$. The intersection of RTA and MRW falls below the point at which all female human capital is employed (where $RTA = MHC$); the wage at

[17] This zero elasticity assumption is consistent with Goldin's (1990) reported estimates of the elasticity of female labour supply in this period.

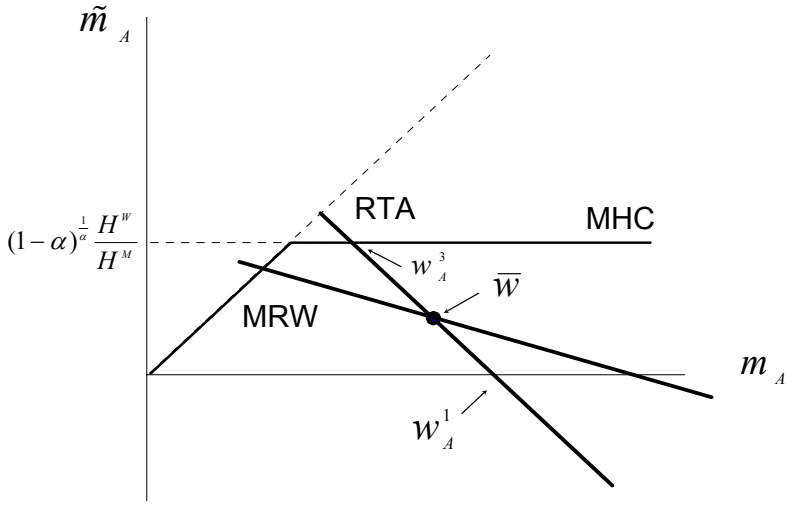

Figure 7: A Phase Two Equilibrium

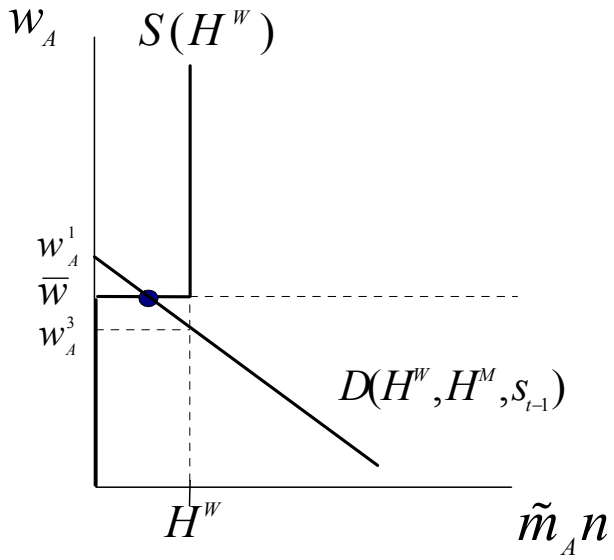

Figure 8: Labour Demand in a Phase Two Equilibrium

which all female human capital is employed, w_A^3, is less than $\overline{w}$. The equilibrium wage is exactly $\overline{w}$, $\chi = \overline{\chi}_t$, and the level of $\widetilde{m}_A$ in phase two is given by the equation:

$$\widetilde{m}_{A,t} = \frac{1-\alpha}{\alpha}\left(\left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{\alpha}{\alpha-\rho}}(\overline{\chi}_t)^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} - \frac{1}{\gamma}s_{t-1} + \overline{\chi}_t\right), \tag{57}$$

In the second phase, with higher levels of human capital, labour demand is greater at every level of the wage (Figure 8). The wage at which all human capital is utilized (w_A^3) is below the reservation wage. The equilibrium wage is equal to $\overline{w}$ and some female human capital remains unemployed.

6.3 Phase Three: Partial Clerical Employment and No Surplus Human Capital

In phase three, the level of human capital is sufficiently high that the wage, when all suppliers who can profitably choose to use the new organizational mode do so, exceeds the reservation wage. All female human capital is employed in sector A. The measure of suppliers using the new organizational mode is exactly $\widetilde{m}_A = \widetilde{n}\frac{H^w}{H^m}$ (with $\widetilde{n} = \left(\frac{1}{1-\alpha}\right)^{\frac{1}{\alpha}}$). Substituting $\widetilde{m}_A = (1-\alpha)^{\frac{1}{\alpha}}\frac{H^w}{H^m}$ into (RTA), the relationship between the level of technology adoption in A is proportional to the level of technology in G according to the equation:

$$m_{A,t} = \left(\frac{1}{1-\alpha}\frac{1-\nu}{\nu}\right)^{\frac{\alpha}{\alpha-\rho}}(\chi_t)^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} - \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\frac{H_t^w}{H_t^m}. \tag{58}$$

The ratio of suppliers using the new organizational mode to all suppliers in the sector, when the economy is in phase three, is:

$$\frac{\widetilde{m}_{A,t}}{m_{A,t}} = \frac{(1-\alpha)^{\frac{1}{\alpha}}\frac{H_t^w}{H_t^m}}{(1-\alpha)^{\frac{\alpha}{\rho-\alpha}}(\chi_t)^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} - \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\frac{H_t^w}{H_t^m}}. \tag{59}$$

The equilibrium in this phase is illustrated in Figures 9 and 10. Again, an increase in human capital has shifted both RTA and MRW up and to the right. If the increase is such that the wage at the point at which RTA intersects MHC is greater than $\overline{w}$, the economy is no longer in phase two. If RTA intersects MRW at a point below the 45° line then the economy is in phase three; $\widetilde{m}_A = (1-\alpha)^{\frac{1}{\alpha}}\frac{H^w}{H^m}$ suppliers use the new organizational mode and all available female human capital is employed. If RTA intersects MRW at a point on the 45° the economy is not in phase three but in phase four.

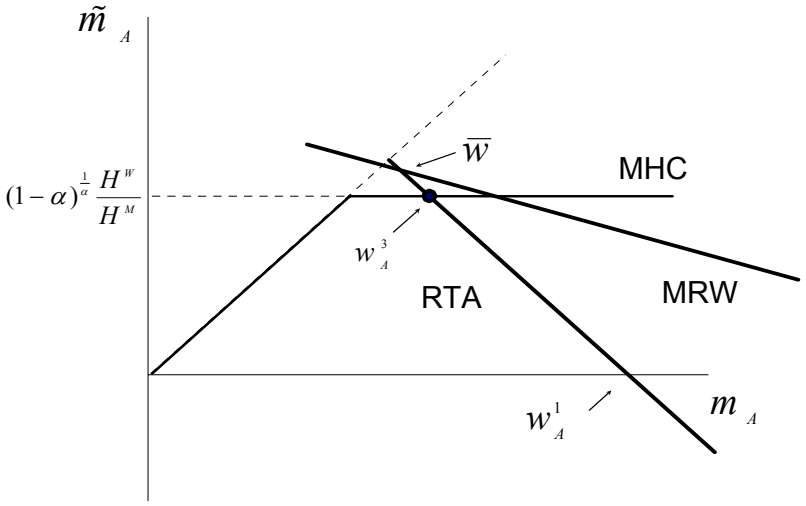

Figure 9: A Phase Three Equilibrium

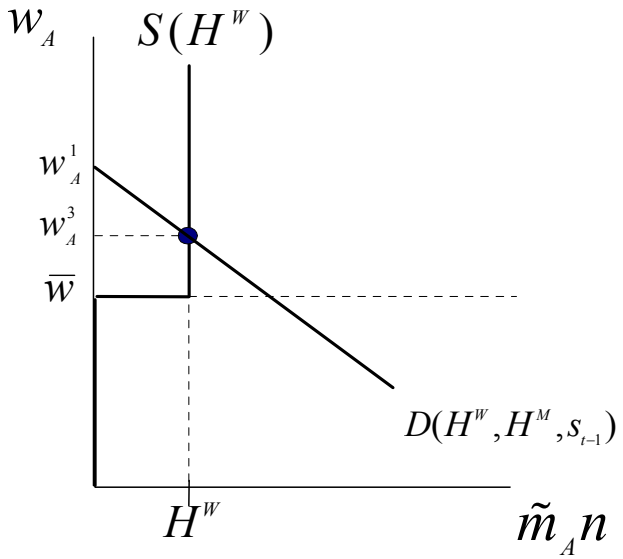

Figure 10: Labour Demand in a Phase Three Equilibrium

In Figure 10, the demand curve for labour moves to the right, as a result of the increase in savings and human capital. The equilibrium wage is above $\overline{w}$ at w_A^3 and $\widetilde{m}_A n = H^w$, and all female human capital is employed.

Proposition 3: *There exists a χ_t^3, such that if $\chi_t^3 > \chi_0$ then i) a phase three equilibrium exists in which all human capital is employed, ii) the variety of intermediates in G is growing proportionally faster than human capital (χ is increasing over time), and iii) in that equilibrium, if female human capital grows rapidly relative to male human capital, the proportion of suppliers that use the new organizational mode increases when human capital increases.*

The third result in proposition 3 depends on the relative growth rates of female and male human capital. Where g_t^w is the growth rate of female human capital in period t, the proportion of suppliers using the new mode is increasing over time is:

$$(1 + g_t^w) > \frac{m_{A,t+1}}{m_{A,t}}. \tag{60}$$

This condition states that the proportion of suppliers using the new organizational mode will increase, as human capital increases, if the curve RTA shifts up and to the right by less than the curve MRW shifts up. If this condition does not hold in phase three, and the rate of technical adoption exceeds the rate of change in the mode of organization, then the economy will stay in phase three; both m_A and $\widetilde{m}_A$ will increase over time but the ratio $\widetilde{m}_A : m_A$ will fall. In order for the economy to reach phase four (where $\widetilde{m}_A = m_A$) the condition in (60) must be satisfied.

Note that for the given wage functions for both male and female human capital it is possible to determine the relationship between the wage in Goods and the wage in Administration in every period. Dividing (27) by (23) and substituting in χ, the ratio of wages in every period is:

$$\frac{w_{A,t}}{w_{G,t}} = (1 - \alpha)^{\frac{1}{\alpha}} \left(\frac{1}{\chi_t}\right) \tag{61}$$

As χ is increasing in phase three (Proposition Three) it must be the case that in every period the wage paid to workers in Administration is falling relative to the wage paid to workers in Goods.

6.4 Phase Four: Full Clerical Employment with No Surplus Human Capital

In phase four all suppliers use the new organizational mode and employ clerical workers: $m_A = \widetilde{m}_A$, $n > \widetilde{n}$ and all female human capital is employed .

Given the relative growth rates of technology in Administration to Goods in (48), the growth rate of the economy in this equilibrium is given by the definition:

$$\Delta \log (m_A) = \Delta \log \chi - \frac{\alpha\rho}{\alpha - \rho + \alpha\rho} \Delta \log H^w + \frac{\alpha\rho}{\alpha - \rho + \alpha\rho} \Delta \log H^m \qquad (62)$$

The relative growth rates in sector A are a function of the relative growth rates of male to female human capital. If female human capital is growing relative to male human capital the level of technology adoption in A is growing slowly relative to the level of technology in G. If, over time, the growth rate of male to female human capital converge, then the growth in the technology in both sectors will also converge.

If male and female human capital grow at the same rate in the long run, the growth path of technology adoption in administration can be described by:

$$\frac{m_{A,t+1}}{m_{A,t}} = (1+g) \left(\frac{m_{A,t}}{m_{A,t-1}} \right)^{\frac{1-\alpha}{\alpha}} \qquad (63)$$

If $\alpha = \frac{1}{2}$ then technology in A will grow at the constant rate of g, the growth rate of human capital. If $\alpha > \frac{1}{2}$, however, the rate of technology adoption will proceed at a faster rate than human capital growth. Therefore, as long as human capital is increasing over time, technology adoption in both A and G will grow at an increasing rate.

Proposition 4: *There exists a χ_t^4 such that for all $\chi_t^4 \geq \overline{\chi}_t$, a phase four equilibrium exists.*

In phase three, the wage rate is a function of both the choice of mode of organization and the rate of technology adoption. An increase in the measure of suppliers using the new organizational mode increases the demand for workers and increases the wage. However, an increase in the measure of suppliers using the new mode of organization also reduces the rate of technology adoption. This reduces output in that sector and puts downward pressure on the wage changes in the organizational mode and the rate of technology adoption has offsetting effects on market wage. In phase four, however, all suppliers use the new organizational mode. An increase in technology has the same effect on the wage as an increase in the measure of suppliers using the new organizational mode in phase three. Without this offsetting effect on wages, the demand curve for female workers is more elastic in phase four than in phase three.

Figure 2.9 and 12 illustrate the equilibrium in phase four. As in phase three, an increase in savings has shifted both RTA and MRW up and to the right. If the growth rate of female human capital relative male human capital is such that RTA intersects the 45° line below MHC with $n = \widetilde{n}$, and, if MRW is everywhere above

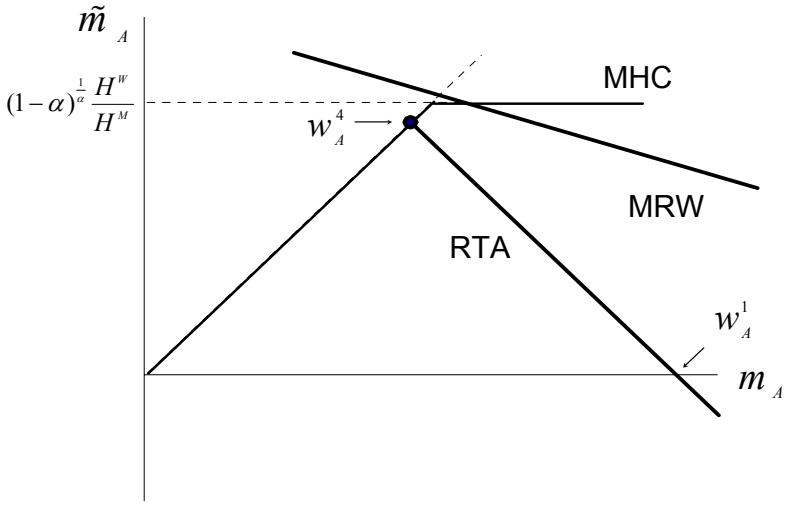

Figure 11: A Phase Four Equilibrium

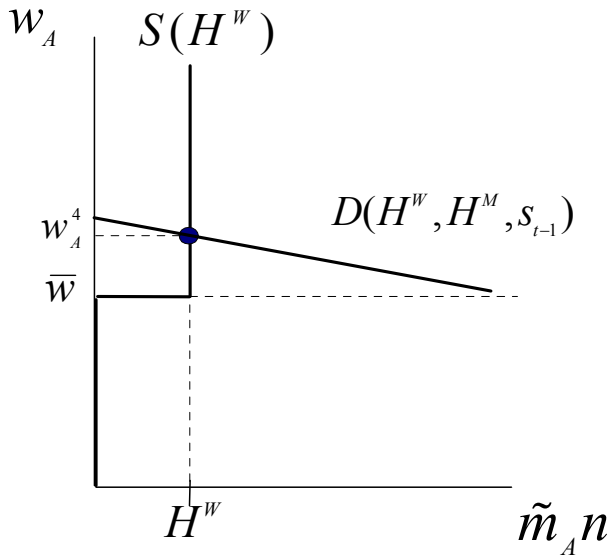

Figure 12: Labour Demand in a Phase Four Equilibrium

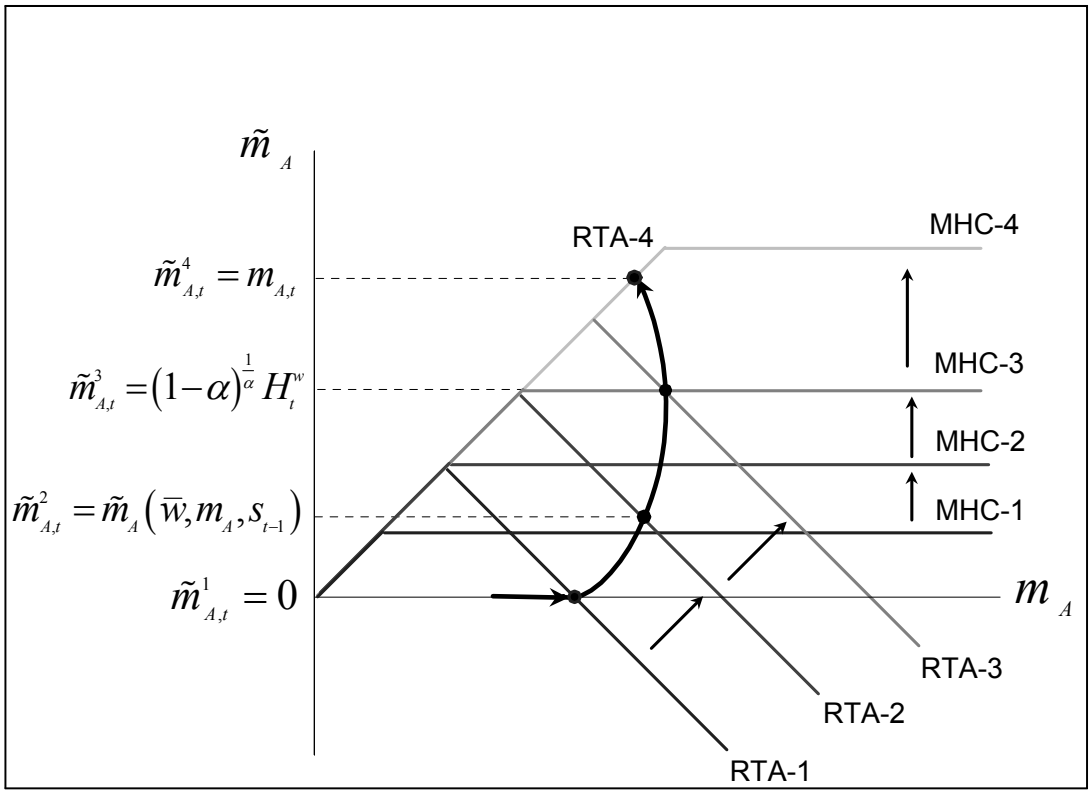

Figure 13: Transitions Through the Phases

RTA, then the economy is in phase four and m_A is determined by the equation in (RTA4). The equilibrium wage in phase four is w^4_A, with $w^4_A > \overline{w}$.

In Figure 12 the demand curve for labour has shifted up and to the left with the increase in human capital. The equilibrium wage is above $\overline{w}$ at w^4_A and $\widetilde{m}_A n = m_A n = H^w$.

6.5 Transitions Through The Phases

Figure 13 illustrates the movement of the economy through the four phases. When human capital is low the economy is in phase one. In phase one, the wage is such that no supplier finds it profitable to use the new organizational mode and hire clerical workers. Given the level of investment in Administration is determined by the relative profitability between Goods and Administrative, and G and A are compliments in the production of the final good, the profitability of investing in the Administration sector is increasing with Goods sector output. Phase one is every point along the

x-axis (with $\widetilde{m}_A = 0$) up to the point at which:

$$m_A\left(s_{t-1}\right) = \frac{1}{\gamma} s_{t-1} - \overline{\chi}_t, \tag{64}$$

and the wage at which a supplier in sector A would be willing to pay workers is equal to exactly the reservation wage.

In phase two, suppliers pay exactly the reservation wage and some of the potential human capital remains unemployed. Where $\overline{\chi}_t$ is increasing in H^m (Proposition One), the level of technology in sector A is increasing if, and only if, the level of savings has increased by more that the increase in $\overline{\chi}_t$. If this is the case, both the level of investment in A and the proportion of suppliers using the new organizational mode is increasing in human capital. If this is not the case, and investment in sector A is decreasing, then it is still the case that the proportion of suppliers using the new organizational mode in sector A is increasing. The demand for female workers is growing in phase two.

Note that the ratio of male to female human capital determines the difference between the market wage in phase one and the market wage in phase three equilibrium. If female human capital is small relative to male human capital, a small increase in *overall* human capital will caused the economy to transition directly from phase one to phase three. If not, and female human capital is close to the level of male human capital, a larger increase in overall human capital will be required to move into phase three.

In phase three, all female human capital is employed in the clerical work force. Individual suppliers are constrained only by the level of labour input they can employ. Both the growth rate of technology adoption and the transition to the new organizational mode is a function of the relative growth rates of male to female human capital. If female human capital is growing slowly relative to male human capital, the level of technology adoption will be rapid but the proportion of suppliers using the new technology will fall over time (M_A increases but the ratio of $\widetilde{m}_A$ to m_A falls). If female human capital is growing rapidly, relative to male human capital, the proportion of suppliers using the new technology will grow over time but the technology in the sector will be falling (M_A decreases and the ratio of $\widetilde{m}_A$ to m_A increases). For a moderate growth rate in female human capital, relative to male human capital, both technology and the proportion of suppliers using the new mode is increasing over time (M_A and the ratio of $\widetilde{m}_A$ to m_A increase). If this is not the case, and the proportion of suppliers using the new mode is falling over time, the economy will stay in phase three despite increases in the skill level of the population. If the growth rates of male and female human capital are converging to a common growth rate, it may be the case that an initial increase in the proportion of suppliers using the new mode is followed by rapid technology growth and a falling proportion of suppliers using the

new mode as the growth rate of female human capital slows relative to that of male human capital.

If phase four is reached, all suppliers have chosen to use the new organizational mode. In this phase, the supply of skilled workers is sufficiently high that suppliers can increase their labour input above $\widetilde{n}$. If n is growing, as human capital increases, the level of technology adoption will slow in that sector. Remembering that Goods and Administration are gross compliments, a sufficiently large increase in labour input in sector A, which increases the level of output per technology, will decrease the level of investment in technologies in that sector. If, over time, male and female human capital growth rates converge, the growth rates of investment in the two sectors will converge as well.

Note that if the increase in output in sector A is sufficient to offset increases in technology adoption, then either the adoption of the new organization mode (in phase three) or the increase in labour input per technology (in phase four) is fueling an increase in technology adoption in the manufacturing sector relative to the administration sector.

7 A Basic Simulation

Given the results of the basic model I have conducted a simple simulation. A full calibration is undertaken in *Calibration Results for a Generalized Model of Female Labour Force Participation* (Adshade, working paper 2004) where a generalized version of the model is developed. While the simulation here cannot be compared to the historic data on the movement of wages and employment, it does give the reader an idea of the relative movements in factors predicted by the model.

The parameter values used for the simulation are given below. The growth rate of male human capital is $q^m = .2$ and female human capital is $q_t^w = 1 - \frac{t}{40}$, so that the growth rates are converging over time. The initial ratio of female to male human capital is $\frac{H^m}{H^w} = 0.096$ and the reservation wage is $\overline{w} = 9.574$.

Figure 14 plots both labour input per technology in Administration and employment in Administration. Vertical lines mark the transition to a new phase, with phase two starting in period 3, phase three in period 4 and phase four in period 14. In every period prior to phase four, $n = 4.605$. In every period in phase four, $n > \widetilde{n}$, and the labour input per technology grows rapidly. Employment growth in Administration is slow initially, but increases rapidly as the proportion of suppliers using the new technology approaches 1. The fastest growth in participation occurs in phase 4

	Basic Model
α	.6
ρ	-2
θ	4
β	.264
ν	.5
γ	1
$\widetilde{n}$	4.6

Table 1: Parameter Values for the Basic Simulation

where it is increasing at a rate of approximately 50% per period. In this simulation participation is increasing due to both a rapid increase in the proportion of suppliers using the new organizational mode (prior to phase four) and a rapid increase in both technology adoption and labour input per technology (phase four).

Figure 15 plots the wage paid to clerical workers and the ratio of the wage in clerical work to the wage in manufacturing. The horizontal line at $w_A \sim 10$ is the reservation wage. In phase one the equilibrium wage falls below $\overline{w}$. In phase two the wage is exactly $\overline{w}$. In every period after phase two the wage is both above $\overline{w}$ and increasing. The relative wage $\left(\frac{w_A}{w_G}\right)$ is falling in every period. Initially clerical workers are paid approximately 16% of the manufacturing wage. At the end of this simulation this has fallen to approximately 8%.

Figure 16 plots the adoption of the new organizational mode relative to technology adoption and Figure 17 plots the logs of m_A and $\widetilde{m}_A$. Both the level of technology and the proportion of suppliers using the new organizational mode is increasing over the simulation. The rate at which suppliers adopt the new mode grows rapidly initially, and converges to the growth rate of technology adoption in phase four (where $\frac{\widetilde{m}_A}{m_A} = 1$).

Figure 18 plots the ratio of output of A and G to the final good, Y. Output in both sectors is growing over the simulation. Output in G, however, as a proportion of total output, is large when no supplier in A uses the new mode organization. As the level of female employment increases the ratio of Administration to output becomes larger than the ratio of Goods to output.

In Figure 19 a large increase in A relative to G coincides with a decrease in the ratio of technologies in A to $G, \left(\frac{M_A}{M_G}\right)$. This suggests that, given the parameter values, a large increase in output in A is the result of a rapid adoption of the new organizational mode rather than a relative increase in technologies. Finally, we see that χ is increasing in every period, as predicted by the model.

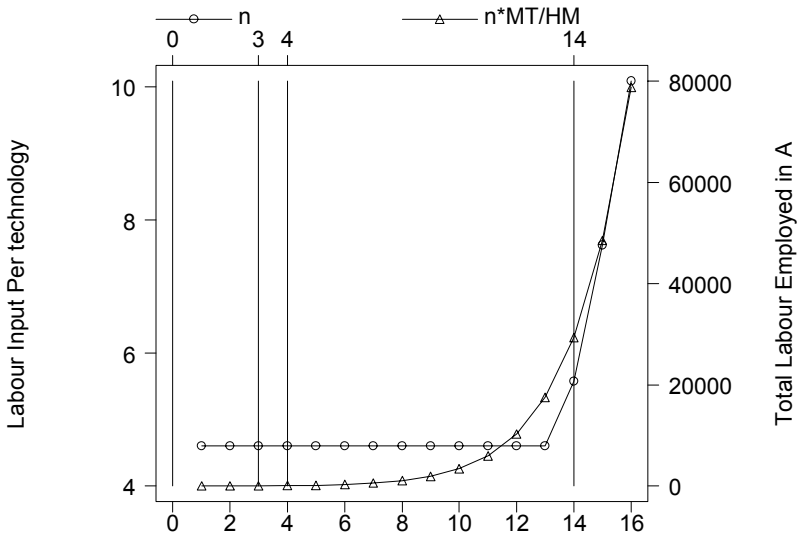

Figure 14: Female Labour Force Participation

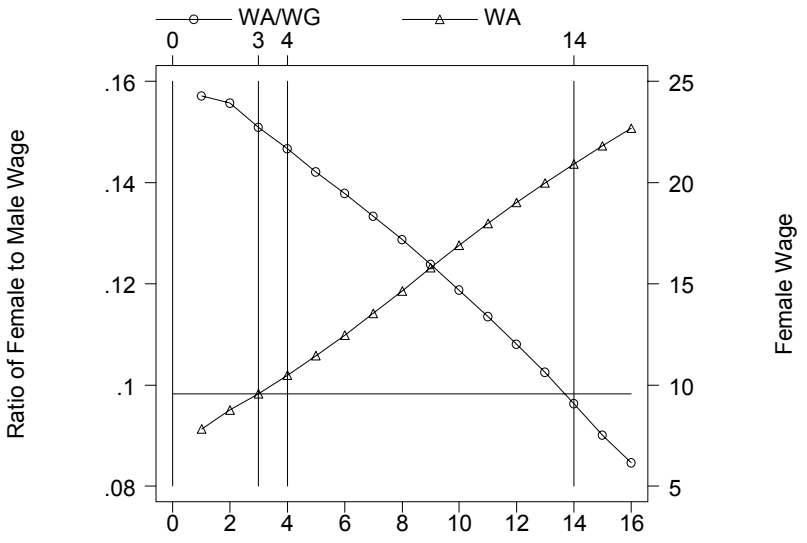

Figure 15: Female Wage and the Ratio of Male to Female Wages

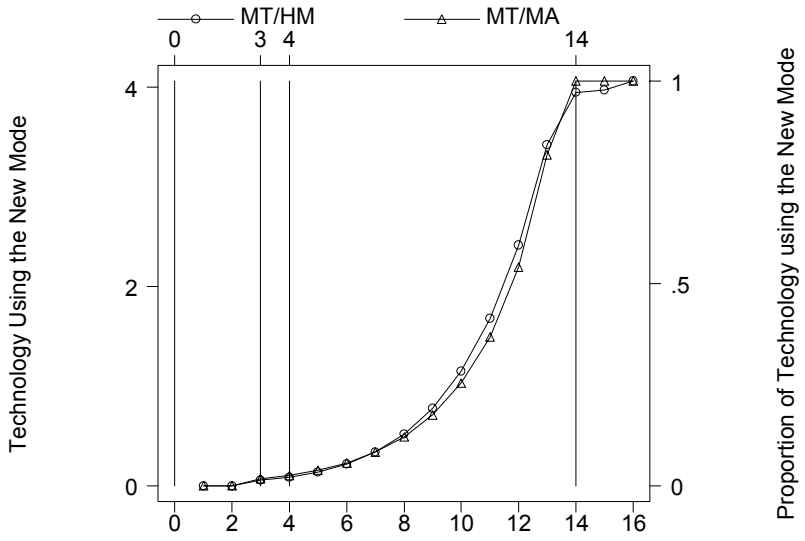

Figure 16: Adoption of the New Mode

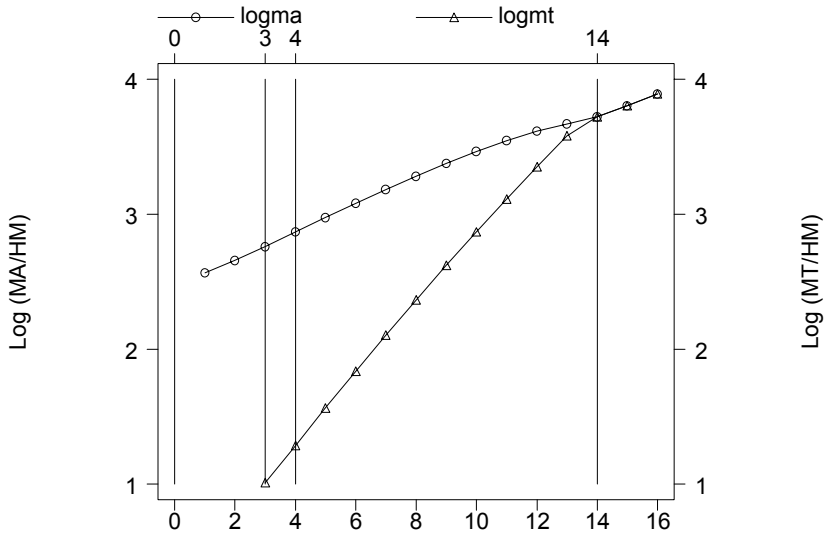

Figure 17: Rates of Change for $\widetilde{m}_A$ and m_A

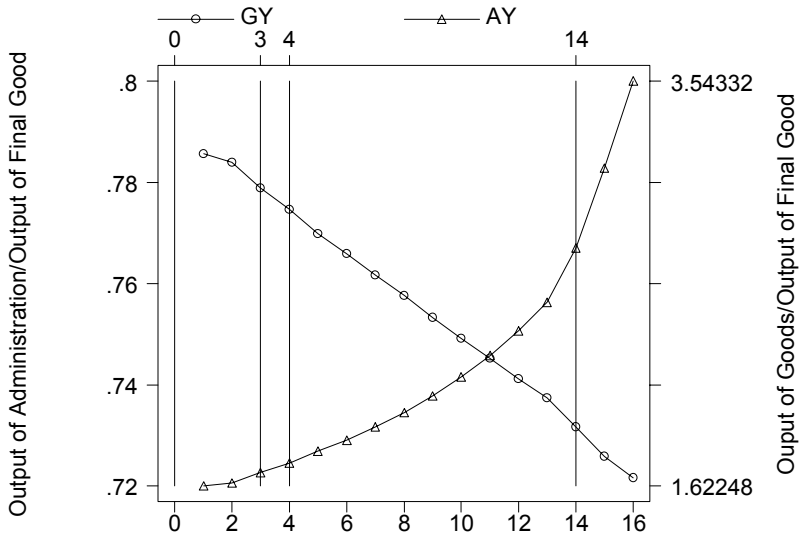

Figure 18: Output of A and G as a Proportion of Total Output.

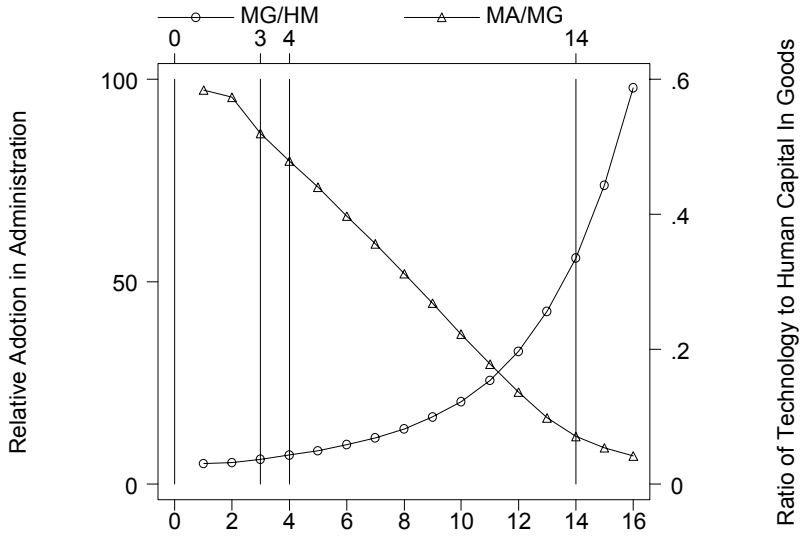

Figure 19: Relative Rates of Technology Adoption

8 Conclusions

A basic model is developed in which an increase in the skill level of the population induces both a change in the way in which production is organized and a re-direction of investment in the adoption of technologies. The analytic results of the model suggest the following:

- Where administrative and manufacturing output are complements in the production of final goods and services, an increase in the skill level of the male workforce is sufficient to induce the adoption of a mode of organization in administration that employes the services of skilled, female workers.
- If demand is such that the market wage is above the reservation wage for female workers, the growth rate of female labour force participation is a function of the growth rate of the skill level of the female workforce.
- As long as some suppliers of administrative services use the old organizational mode, the number of workers per technology will stay low as suppliers compete for the limited number of available skilled workers. If the availability of skilled workers is sufficiently high that all suppliers use the new organizational mode, the number of workers per technology increases.
- If the skill level of the female workforce grows rapidly, relative to male human capital, the proportion of suppliers of administrative services who use the new mode of organization grows over time.
- If either the adoption of the new mode of organization or the increase in labour input per technology is sufficiently slow relative to the growth in savings, then investment in the adoption of administrative technologies increases over time.

One important extension to this model would be to allow the price of the substitutes to household produced goods and services to change over time. Greenwood et al. (2001) have suggested that the falling cost of home production technologies (specifically in the form of semi-durables) was an important factor in the decision of many women to join the waged work force. Empirical evidence suggests that this had its biggest effect in the post-W.W.II period. If changes in technology and organizational structure were pulling women into the waged workforce in the first half of the century, it is possible that the arrival of home production technologies were effectively pushing women out of the home, and into the work force, in the second half. In fact, the early increase in female labour force participation discussed in this paper suggests that the demand for the substitutes to home production were growing over time. It is perhaps the case that this early increase in the demand for home production technologies induced an increase in the supply of those technologies. If

investment was such that the price of these technologies was falling relative to other consumption goods, then the reservation wage to female labour was effectively falling over time. By developing a model of female labour force participation with endogenous technical change in the provision of the substitutes to home production, it might be possible to form a complete picture of the movement of women into the waged workforce over the twentieth century.

9 Definition of Variables used in the model

c	Consumption of market produced goods and services
h	Consumption of household produced goods and services
I	Total household income
S	Savings
Y	Final good output
A	Administration output
O	Goods output
x_A	Output of a supplier in administration sector
x_G	Output of a supplier in goods sector
n	Labour input to a supplier in administration
l	Labour input to a supplier in goods
m_A	Measure of technology/suppliers in administration
m_G	Measure of technology/suppliers in goods
$\widetilde{m}_A$	Measure of technology/suppliers in administration using new mode
H^m	Male human capital
H^w	Female human capital
L_u	Stock of unskilled workers (for men (L_u^m) and women (L_u^w))
L_s	Stock of skilled workers (for men (L_s^m) and women (L_s^w))
$\overline{w}$	Reservation wage

All lower case technology variables are deflated by the level of male human capital.

A Proof of Lemma 1

In order for the wage to be decreasing in $\widetilde{m}_A$ it must be the case that if overall investment in A, determined by the level of m_A, is a decreasing function of $\widetilde{m}_A$, then the wage is an increasing function of m_A. Where the slope of the labour demand curve is:

$$\frac{dw_{A,t}}{d\widetilde{m}_{A,t}} = \frac{dw_{A,t}}{dm_{A,t}} \frac{dm_{A,t}}{d\widetilde{m}_{A,t}} < 0, \tag{65}$$

the derivative of $\widetilde{m}_A$ with respect to $m_{A,t}$, where $\psi = \frac{1}{1-\alpha}\frac{1-\nu}{\nu}$, is:

$$\frac{d\widetilde{m}_{A,t}}{dm_{A,t}} = \left(n_t^{\alpha} - 1\right)^{-1} \left(-\frac{\alpha - \rho + \alpha\rho}{\alpha - \rho} \psi^{\frac{\rho}{\alpha-\rho}} \chi_t^{\frac{\alpha\rho}{\alpha-\rho}} - 1 \right). \tag{66}$$

With $\rho < 0$, the condition that $\frac{dm_{A,t}}{d\widetilde{m}_{A,t}} < 0$ is simply $\chi_t > 0$.

Writing $w_{A,t}$ in (27) as a function of $m_{A,t}$:

$$w_A\left(m_{A,t}\right) = \frac{\theta\alpha\left(1-\alpha\right)}{n^{1-\alpha}} \nu H^{m\frac{1-\alpha}{\alpha}} \left(\nu + \left(1-\nu\right)\psi^{\frac{\rho}{\alpha-\rho}} \left(\frac{1}{\gamma} s_{t-1} - m_{A,t} \right)^{\frac{\alpha\rho}{\alpha-\rho}} \right)^{\frac{1-\rho}{\rho}} \left(\frac{1}{\gamma} s_{t-1} - m_{A,t}^{\frac{\alpha\rho}{\alpha-\rho}} \right)^{\frac{1-2\alpha}{\alpha}} \tag{67}$$

and taking derivative with respect to $m_{A,t}$:

$$\begin{aligned} \frac{dw_{A,t}}{dm_{A,t}} &= \frac{\theta\left(1-\alpha\right)\nu}{n^{1-\alpha}} H^{m\frac{1-\alpha}{\alpha}} \left(2\alpha - 1\right) \left(\nu + \left(1-\nu\right)\left(\psi\chi^{\alpha}\right)^{\frac{\rho}{\alpha-\rho}} \right)^{\frac{1-\rho}{\rho}} \chi^{\frac{1-2\alpha}{\alpha}-1} \\ &\quad - \frac{\theta\left(1-\alpha\right)\nu}{n^{1-\alpha}} H^{m\frac{1-\alpha}{\alpha}} \frac{\alpha^2\left(1-\rho\right)}{\alpha-\rho} \left(1-\nu\right)\psi^{\frac{\rho}{\alpha-\rho}} \chi^{\frac{\alpha\rho}{\alpha-\rho}-1} \left(\nu + \left(1-\nu\right)\left(\psi\chi^{\alpha}\right)^{\frac{\rho}{\alpha-\rho}} \right)^{\frac{1-\rho}{\rho}-1} \chi^{\frac{1-2\alpha}{\alpha}} \end{aligned} \tag{68}$$

the condition that $\frac{dw_{A,t}}{dm_{A,t}} > 0$ is: $\alpha > \frac{1}{2}$ and $\chi_t > \chi_0$, where:

$$\chi_0 = \left(\left(\frac{\alpha}{2\alpha - 1} \frac{\alpha\left(1-\rho\right)}{\alpha-\rho} \frac{\left(1-\nu\right)}{\nu} - 1 \right) \left(\frac{1}{1-\alpha} \frac{1-\nu}{\nu} \right)^{\frac{\rho}{\alpha-\rho}} \right)^{\frac{\alpha-\rho}{\alpha\rho}}. \tag{69}$$

For all $\chi_t > \chi_0$ and $\alpha > \frac{1}{2}$, the wage paid in administration is an increasing function of the rate of innovation in that sector and the labour demand curve for clerical workers is downward sloping.

B Proof of Proposition 2:

B.1 Existence of an equilibrium in which no suppliers use the new organizational mode:

The level of χ in phase one is χ_t^1 which solves the equation:

$$\psi^{\frac{\rho}{\alpha-\rho}}\left(\chi_t^1\right)^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} = \frac{1}{\gamma}s_{t-1} - \chi_t^1. \tag{70}$$

Where the term on the left hand side is positively sloped and concave when $\rho < 0$, and the right hand side is linear in $\overline{\chi}_t$, there is a unique level of χ_t^1 that solves this equation in phase one.

When $\chi = \chi_t^1$, the wage in A is:

$$w_A\left(\chi_t^1\right) = \frac{1-\alpha}{n_t^{1-\alpha}}\nu\theta H_t^{m\frac{1-\alpha}{\alpha}}\alpha\left(\nu + (1-\nu)\left(\psi\chi_t^{1\alpha}\right)^{\frac{\rho}{\alpha-\rho}}\right)^{\frac{1-\rho}{\rho}}\left(\chi_t^1\right)^{\frac{1-2\alpha}{\alpha}} \tag{71}$$

The term in the right hand side of the wage equation is strictly decreasing in χ, given the conditions in Lemma 1. A phase one equilibrium exists for all levels of $\chi_t^1 > \overline{\chi}_t$ (where $\overline{\chi}_t$ is the level of χ when $w_{A,t} = \overline{w}$), and $w_A\left(\chi_t^1\right) < \overline{w}$, and is unique for all levels of $\chi_{t=0} > 0$. If $\chi_t^1 < \overline{\chi}_t$ then $w_{A,t} > \overline{w}$, and the economy is not in phase one, at least one producer will choose to adopt the new organizational mode.

B.2 The variety of intermediates is growing over time:

Equations (28) and (29) simultaneously determine the ratio of intermediates in A to intermediates in G. In phase one, where $W_t = H_{m,t}w_{G,t}$:

$$m_{A,t+1} = \frac{\beta}{(1+\beta)\gamma}w_{G,t} - \chi_{t+1} \tag{72}$$

Letting $\frac{\beta}{(1+\beta)\gamma} = \Omega$ and:

$$w_{G,t} = \theta\alpha\nu\left(v + (1-\nu)\left(\psi\chi^{\alpha}\right)^{\frac{\rho}{\alpha-\rho}}\right)^{1-\rho}H_t^{m\frac{1-\alpha}{\alpha}}\chi_t^{\frac{1-\alpha}{\alpha}}, \tag{73}$$

the ratio of intermediates in A to G, when savings equals investment, is:

$$\frac{m_{A,t+1}}{\chi_{t+1}} = \Omega\alpha\nu\theta\frac{H_t^{m\frac{1}{\alpha}}}{H_{t+1}^m}\frac{\chi_t^{\frac{1-\alpha}{\alpha}}}{\chi_{t+1}}\left[\nu + \left(\psi\chi_t^{\alpha}\right)^{\frac{\rho}{\alpha-\rho}}\right]^{\frac{1-\rho}{\rho}} - 1. \tag{74}$$

The ratio of intermediates in Administration to intermediates in Goods when profits are equal across sectors is:

$$\frac{m_{A,t+1}}{\chi_{t+1}} = \psi^{\frac{\alpha}{\alpha-\rho}} \chi_{t+1}^{\frac{\alpha\rho}{\alpha-\rho}} \tag{75}$$

which is decreasing in χ when $\rho < 0$.

Equating (74) and (75) and solving for χ_{t+1}:

$$\psi^{\frac{\alpha}{\alpha-\rho}} \chi_{t+1}^{\frac{\alpha-\rho+\alpha\rho}{\alpha\rho}} + \chi_{t+1} = \Omega \frac{\alpha\nu\theta H_t^{m\frac{1}{\alpha}} \chi_t^{\frac{1-\alpha}{\alpha}}}{H_{t+1}^m} \left[\nu + (\psi\chi_t^{\alpha})^{\frac{\rho}{\alpha-\rho}}\right]^{\frac{1-\rho}{\rho}} \tag{76}$$

If H^m grows at constant rate g^m, then lagging (76) by one period, and dividing period $t+1$ by period t yields the ratio:

$$\frac{\psi^{\frac{\alpha}{\rho-\alpha}} \chi_{t+1}^{\frac{\alpha-\rho+\alpha\rho}{\alpha\rho}} + \chi_{t+1}}{\psi^{\frac{\alpha}{\rho-\alpha}} \chi_t^{\frac{\alpha-\rho+\alpha\rho}{\alpha\rho}} + \overline{\chi}_t} = (1+g^m)^{\frac{1-\alpha}{\alpha}} \left(\frac{\chi_t}{\chi_{t-1}}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{\nu + (1-\nu)\psi\chi_t^{\frac{\alpha\rho}{\alpha-\rho}}}{\nu + (1-\nu)\psi\chi_{t-1}^{\frac{\alpha\rho}{\alpha-\rho}}}\right)^{\frac{1-\rho}{\rho}} \tag{77}$$

If $\rho < 0$, the derivative $\left(\chi_{t-1}\right)^{\frac{1-\alpha}{\alpha}} \left(\nu + (1-\nu)\psi\chi_{t-1}^{\frac{\alpha\rho}{\alpha-\rho}}\right)^{\frac{1-\rho}{\rho}}$ with respect to χ_{t-1} :

$$\begin{aligned} &= \frac{1-\alpha}{\alpha} \chi_{t-1}^{\frac{1-2\alpha}{\alpha}} \left(\nu + (1-\nu)\psi\chi_{t-1}^{\frac{\alpha\rho}{\alpha-\rho}}\right)^{\frac{1-\rho}{\rho}} \\ &+ \frac{1-\rho}{\rho} \frac{\alpha\rho}{\alpha-\rho} \chi_{t-1}^{\frac{1-\alpha}{\alpha}} (1-\nu)\psi\chi_{t-1}^{\frac{\alpha\rho}{\alpha-\rho}-1} \left(\nu + (1-\nu)\psi\chi_{t-1}^{\frac{\alpha\rho}{\alpha-\rho}}\right)^{\frac{1-2\rho}{\rho}} \end{aligned} \tag{78}$$

is greater than zero for all values of $\chi_t > 0$.

If $\chi_t > \chi_{t-1} > 0$, then substituting χ_t in for χ_{t-1} in (77) yields the inequality:

$$\frac{\psi^{\frac{\alpha}{\alpha-\rho}} \chi_{t+1}^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} + \chi_{t+1}}{\psi^{\frac{\rho}{\alpha-\rho}} \chi_t^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} + \chi_t} > (1+g^m)^{\frac{1-\alpha}{\alpha}}$$

or, as $(1+g^m)^{\frac{1-\alpha}{\alpha}} > 1$:

$$\psi^{\frac{\rho}{\alpha-\rho}} \chi_{t+1}^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} + \chi_{t+1} > \psi^{\frac{\rho}{\alpha-\rho}} (\chi_t)^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} + \chi_t. \tag{79}$$

The derivative of the left hand side (LHS) of this inequality with respect to χ_{t+1} :

$$\frac{dLHS}{d\chi_{t+1}} = \frac{\alpha-\rho+\alpha\rho}{\alpha-\rho} \psi^{\frac{\rho}{\alpha-\rho}} \chi_{t+1}^{\frac{\alpha\rho}{\alpha-\rho}} + 1 \tag{80}$$

is greater than zero for all $\chi_{t+1} > 0$. If follows that if $\chi_t > \chi_{t-1}$,then $\chi_{t+1} > \chi_t$.

B.3 The growth rate of intermediates in Administration is slow relative to that in Goods:

If the ratio of the ratio of intermediates in A to intermediates in G in (75) is decreasing in χ_t, it must be the case that when $\widetilde{m}_{A,t} = 0$, and as χ grows over time, the proportion of suppliers of intermediates to Administration to suppliers of intermediates to Goods also grows over time.

C Proof of Proposition 3:

C.1 Existence of an equilibrium in which all human capital is employed:

The level of χ in phase one is χ_t^3 which solves the equation:

$$\psi^{\frac{\rho}{\alpha-\rho}}\left(\chi_t^3\right)^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} = \frac{1}{\gamma}s_{t-1} - \chi_t^3 + \left(\frac{\alpha}{1-\alpha}\right)(1-\alpha)^{\frac{1}{\alpha}}\left(\frac{H_t^w}{H_t^m}\right) \tag{81}$$

The term on the left hand side of the above is negatively sloped and convex if $\rho < 0$. Where the right hand side is linear in χ, there is a unique χ_t^3 that solves the equation for all $\chi > 0$.

When $\chi = \chi_t^3$, the wage in A is:

$$w_A\left(\chi_t^3\right) = \frac{1-\alpha}{n_t^{1-\alpha}}\theta\nu {H_t^m}^{\frac{1-\alpha}{\alpha}}\alpha\left(\nu + (1-\nu)\left(\psi\chi_t^{3\alpha}\right)^{\frac{\rho}{\alpha-\rho}}\right)^{\frac{1-\rho}{\rho}}\left(\chi_t^3\right)^{\frac{1-2\alpha}{\alpha}} \tag{82}$$

Where wage equation is strictly decreasing in χ given the conditions in the lemma, a phase three equilibrium exists if both $\chi_t^3 \leq \overline{\chi}_t$ (where $\overline{\chi}_t$ is the level of χ when $w_{A,t} = \overline{w}$) such that $w_A\left(\chi_t^3\right) > \overline{w}$ and if $m_{A,t} > (1-\alpha)^{\frac{1}{\alpha}}\frac{H_t^w}{H_t^m}$ (i.e. $\frac{\widetilde{m}_{A,t}}{m_{A,t}} < 1$). If either $\chi_t^3 > \overline{\chi}_t$ and $w_A\left(\chi_t^3\right) < \overline{w}$ or $m_A \leq (1-\alpha)^{\frac{1}{\alpha}}\frac{H_t^w}{H_t^m}$ the economy is not in phase three.

C.2 The variety of intermediates in Goods is growing proportionally faster than human capital:

Equations (28) and (29) simultaneously determine the ratio of intermediates in A to intermediates in G. Where:

$$m_{A,t+1} = \frac{\beta}{(1+\beta)\gamma}\left(H_t^m w_{G,t} + H_t^w w_{A,t} - H_t^w\overline{w}\right) - \chi_{t+1} \tag{83}$$

and where:

$$w_{G,t} = \alpha\theta\nu {H_t^m}^{\frac{1-\alpha}{\alpha}} \left[\nu + (1-\nu)\left(\psi\chi_t^{\alpha}\right)^{\frac{\rho}{\alpha-\rho}}\right]^{\frac{1-\rho}{\rho}} {\chi_t}^{\frac{1-\alpha}{\alpha}} \tag{84}$$

$$w_{A,t} = (1-\alpha)^{\frac{1}{\alpha}} \chi_t^{-1} w_{G,t} \tag{85}$$

the ratio of intermediates is:

$$\frac{m_{A,t+1}}{\chi_{t+1}} = \Omega \frac{1}{H_{t+1}^m \chi_{t+1}} \alpha\nu\theta \left(H_t^m \chi_t\right)^{\frac{1-\alpha}{\alpha}} \left[\nu + (1-\nu)\left(\psi\chi_t^{\alpha}\right)^{\frac{\rho}{\alpha-\rho}}\right]^{\frac{1-\rho}{\rho}} \tag{86}$$

$$\times \left(H_t^m + \frac{H_t^w}{\chi_t}(1-\alpha)^{\frac{1}{\alpha}}\right) \tag{87}$$

$$-\Omega \frac{1}{H_{t+1}^m \chi_{t+1}} H_t^w \overline{w} - 1 \tag{88}$$

Substituting χ_t into (RTA), the ratio of intermediates in A to intermediates in G when profits are equal and when the economy is in phase three is:

$$\frac{m_{A,t+1}}{\chi_{t+1}} = \psi^{\frac{\rho}{\alpha-\rho}} \chi_{t+1}^{\frac{\alpha\rho}{\alpha-\rho}} - \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \frac{H_{t+1}^w}{H_{t+1}^m \chi_{t+1}} \tag{89}$$

Equating (86) and (89) and solving for χ_{t+1}:

$$\psi^{\frac{\rho}{\alpha-\rho}} \chi_{t+1}^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} + \chi_{t+1} = \Omega\alpha\nu\theta \frac{{H_t^m}^{\frac{1}{\alpha}}}{H_{t+1}^m} {\chi_t}^{\frac{1-\alpha}{\alpha}} \left[\nu + (1-\nu)\left(\psi\chi_t^{\alpha}\right)^{\frac{\rho}{\alpha-\rho}}\right]^{\frac{1-\rho}{\rho}} \tag{90}$$

$$\times \left(1 + \frac{H_t^w}{\chi_t H_t^m}(1-\alpha)^{\frac{1}{\alpha}}\right) \tag{91}$$

$$-\Omega \frac{H_t^w}{H_{t+1}^m} \overline{w} + \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \frac{H_{t+1}^w}{H_{t+1}^m} \tag{92}$$

Lagging by one period,and dividing period $t+1$ by period t, the derivative of the denominator of the resulting right hand side with respect to χ_{t-1} is:

$$= \alpha\theta {H_{t-1}^m}^{\frac{1}{\alpha}} \left(\frac{\alpha(1-\rho)}{\alpha-\rho}(1-\nu)\psi^{\frac{\rho}{\alpha-\rho}} \chi_{t-1}^{\frac{\alpha\rho-\alpha-\rho}{\alpha-\rho}} \Upsilon_{t-1}^{\frac{1-2\rho}{\rho}} {\chi_{t-1}}^{\frac{1-\alpha}{\alpha}} \left(+\frac{H_{t-1}^w}{\chi_{t-1} H_{t-1}^m}(1-\alpha)^{\frac{1}{\alpha}}\right)\right)$$

$$+\alpha\theta {H_{t-1}^m}^{\frac{1}{\alpha}} \left(\Upsilon_{t-1}^{\frac{1-\rho}{\rho}} \chi_{t-1}^{\frac{1-2\alpha}{\alpha}} \left(\frac{1-\alpha}{\alpha} + \frac{H_{t-1}^w}{\chi_{t-1} H_{t-1}^m}(1-\alpha)^{\frac{1}{\alpha}} \frac{1-2\alpha}{\alpha}\right)\right) \tag{93}$$

Where $\Upsilon_t = \left[\nu + (1-\nu)\left(\psi\chi_t^{\alpha}\right)^{\frac{\rho}{\alpha-\rho}}\right]$. If $\rho < 0$ and $\alpha > \frac{1}{2}$ this condition is greater than zero for all $\chi_t > \chi_0$ determined in the Lemma 1.

Given the above conditions, if $\chi_{t-1} < \chi_t$ then substituting χ_t in for χ_{t-1} yields the inequality:

$$\frac{\psi^{\frac{\rho}{\alpha-\rho}}\left(\frac{1}{\chi_{t+1}}\right)^{\frac{\alpha-\rho+\alpha\rho}{\rho-\alpha}}+\chi_{t+1}}{\psi^{\frac{\rho}{\alpha-\rho}}\left(\frac{1}{\chi_t}\right)^{\frac{\alpha-\rho+\alpha\rho}{\rho-\alpha}}+\chi_t} > (1+g)^{\frac{1-\alpha}{\alpha}} > 1 \tag{94}$$

Which, is greater than zero for all values of $\chi_t > 0$ when $\rho < 0$. If follows that if $\chi_t > \chi_{t-1} > \chi_0$,then $\chi_{t+1} > \chi_t$.

C.3 The proportion of suppliers using the new mode of organization increases for a sufficiently high growth rate of female human capital:

The ratio of suppliers using the new mode to suppliers using the old mode is:

$$\frac{\widetilde{m}_{A,t}}{m_{A,t}} = \frac{(1-\alpha)^{\frac{1}{\alpha}}\frac{H_t^w}{H_t^m}}{\psi^{\frac{\rho}{\alpha-\rho}}\chi_t^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} - \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\frac{H_t^w}{H_t^m}}, \tag{95}$$

which is decreasing in χ_t.

If male human capital grows at rate g^m and female human capital grows at rate g_t^w then the proportion of producer using the new organizational mode is growing in every period if:

$$\frac{\frac{\widetilde{m}_{A,t+1}}{m_{A,t+1}}}{\frac{\widetilde{m}_{A,t}}{m_{A,t}}} = \frac{(1+g_t^w)}{(1+g^m)}\frac{\psi^{\frac{\rho}{\alpha-\rho}}\chi_t^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} - \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\frac{H_t^w}{H_t^m}}{\psi^{\frac{\rho}{\alpha-\rho}}\chi_{t+1}^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} - \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\frac{(1+g^w)}{(1+g^m)}\frac{H_t^w}{H_t^m}} > 1 \tag{96}$$

Which simplifies to:

$$\frac{(1+g_t^w)}{(1+g^m)} > \left(\frac{\chi_{t+1}}{\chi_t}\right)^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}} \tag{97}$$

Solving (RTA) for $\chi_t^{\frac{\alpha-\rho+\alpha\rho}{\alpha-\rho}}$ and substituting in the above this condition simplifies to:

$$(1+g_t^w) > \frac{m_{A,t+1}}{m_{A,t}} \tag{98}$$

If the rate at which technology in sector A is growing is slower than the rate at which female human capital is growing then the proportion of suppliers using the new organizational mode will increase with human capital.

D Proof of Proposition 4

In phase four, m_A is proportional to savings according to the condition:

$$m_{A,t} = \frac{\left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\alpha-\rho}}}{\left(\left(\frac{H_t^m}{H_t^w}\right)^{\frac{\alpha\rho}{\alpha-\rho+\alpha\rho}} + \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\alpha-\rho}}\right)} \frac{1}{\gamma} s_{t-1} \tag{99}$$

Where $\chi_t^4 = \frac{1}{\gamma} s_{t-1} - m_{A,t}$, the wage in phase four is:

$$w_A\left(\chi^4\right) = \alpha\nu\theta H_t^{m\frac{1}{\alpha}} \left(\frac{H_t^w}{H_t^m}\right)^{\frac{\alpha\rho}{\alpha-\rho+\alpha\rho}} \left(\nu + (1-\nu)\left(\frac{1-\nu}{\nu}\right)^{\frac{\rho}{\alpha-\rho}} \left(\frac{H_t^m}{H_t^w}\right)^{\frac{\alpha\rho}{\alpha-\rho+\alpha\rho}}\right)^{1-\rho} \left(\chi_t^4\right)^{\frac{1-\alpha}{\alpha}}. \tag{100}$$

If $w_A\left(\chi^4\right) < \overline{w}$ the economy is not in phase four. If $w_A\left(\chi^4\right) \geq \overline{w}$ the economy is not in phase four if $m_A > (1-\alpha)^{\frac{1}{\alpha}} \frac{H_t^w}{H_t^m}$ as condition on the division of labour in (49) is not met. If $m_A \leq (1-\alpha)^{\frac{1}{\alpha}} \frac{H_t^w}{H_t^m}$ and if $w_A\left(\chi^4\right) > \overline{w}$ the economy is in phase four.

E Solutions when $\alpha > \rho > 0$ and $\rho > \alpha$.

Although the equilibrium has been examined when $\rho < 0$, the main results of the paper when both $\alpha > \rho > 0$ and $\rho > \alpha$. The main difference when $\rho > 0$ is that the relative growth rates between sectors varies. Consider equation (56) that determines the relative growth rates of m_G and m_A. If $\rho < 0$, as we have assumed, A and G are gross compliments and both increase as savings increases. If $0 < \rho < \alpha$, however, m_A will grow proportionally faster than m_G. If $\frac{\alpha}{1-\alpha} > \rho > \alpha$, positive growth in m_G will be accompanied by negative growth in m_A. In either case the demand for female workers continues to increase over time.

In the proof to Lemma 1, the proof that $\frac{d\widetilde{m}_{A,t}}{dm_{A,t}} < 0$ when $\rho < 0$ required that $\chi_t > 0$. If $\rho > 0$ this condition is $\chi_t > \widehat{\chi}$, where:

$$\widehat{\chi} = \left(\frac{\alpha - \rho + \alpha\rho}{\rho - \alpha} (1-\alpha)^{\frac{\alpha}{\rho-\alpha}}\right)^{\frac{\rho-\alpha}{\alpha\rho}} \tag{101}$$

In the proof to Proposition 2 the ratio of intermediates in A to intermediates in G is decreasing in χ if $\rho < 0$, increasing if both $\rho > 0$ and $\rho < \alpha$ and decreasing if $\rho > \alpha$.The result that χ is increasing as human capital increases requires that $\chi_{t=0} > \max\left\{\widehat{\chi}, \widetilde{\chi}\right\}$ where:

$$\widetilde{\chi} = \left(\left(\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}\frac{\alpha\rho}{\rho-\alpha} - 1\right)\frac{(1-\nu)}{\nu}(1-\alpha)^{\frac{\rho}{\rho-\alpha}}\right)^{\frac{\rho-\alpha}{\alpha\rho}} \tag{102}$$

In the proof to proposition 3, the condition that the ratio of intermediates in G to male human capital requires that the initial value of χ meets the condition $\chi_{t=0} > \max\{\widehat{\chi}, \widetilde{\chi}\}$ when $\overline{w} > 1$ and $\chi_{t=1} > \max\{\widehat{\chi}, \chi^*\}$ otherwise:

$$\chi^* > \widetilde{\chi} \left(\frac{\left(H_{t-1}^m + H_{t-1}^w \left(\frac{1}{\chi_{t-1}} - \overline{w} \right) \right)}{\left(H_{t-1}^m + H_{t-1}^w \left(\frac{1}{\chi_{t-1}} - \overline{w} \right) - \frac{\alpha}{1-\alpha} H_{t-1}^w \chi_{t-1} \left(1 - \overline{w} \right) \right)} \right)^{\frac{\rho-\alpha}{\alpha\rho}}.$$

References

[1] Acemoglu, Daron. "Technical Change, Inequality and the Labour Market." Working Paper. (2001)

[2] Acemoglu, Daron. "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality." Quarterly Journal of Economics, No. 113. (November 1998)

[3] Acemoglu, Daron. "Technical Change, Inequality and the Labour Market." Working Paper. (2001)

[4] Adshade, Marina. "Calibration Results of a Generalized Model of Female Labour Force Participation in an Era of Technological and Organization Change." Working Paper. (2004)

[5] Beaudry, Paul; Green, David."What is Driving US and Canadian Wages: Exogenous Technical Change or Endogenous Choice of Technique?" National Bureau of Economic Research Working Paper, No. 6853 (December 1998) Forthcoming American Economic Review..

[6] Costa, Dora L. "From Mill Town To Board Room: The Rise of Women's Paid Labor." NBER Working Paper Series, No. 7608. (March 2000)

[7] Dixit, A. Stiglitz, J."Monopolistic Competition and Optimum Product Diversity." American Economic Review, Vol. 73 No. 3. (June 1977)

[8] Ethier, Wilfred J.. "Internationally Decreasing Costs and World Trade." Journal of International Economics, No. 9. (February 1979)

[9] Freeman, Richard; Schettkat, Ronald. "Low Wage Services: Interpreting the U.S.-German Difference." NBER Working Paper Series, No. 7611. (March 2000).

[10] Katz, Lawrence; Goldin, Claudia. "The Decline of Noncompeting Groups: Changes in the Premium to Education, 1890 to 1940." NBER Working Paper No. 5202. (1995)

[11] Katz, Lawrence; Goldin, Claudia."The Returns to Skill across the Twentieth Century United States." National Bureau of Economic Research Working Paper, No. 7126 (May 1999).

[12] Goldin, Claudia. "Understanding the Gender Gap: An Economic History of American Women." Oxford: Oxford University Press. (1990)

[13] Goldin, Claudia. "Appendix to: "How America Graduated From High School, 1910 to 1960", Construction of State-Level Secondary School Data." National Bureau of Economic Research Working Paper Series on Historical Factors in Long Run Growth: 57. (June,1994)

[14] Goldin, Claudia. "The U-Shaped Female Labor Force Function in Economic Development and Economic History." In T. Paul Schultz ed., *Investment in Women's Human Capital*. Chicago: The University of Chicago Press. (1995)

[15] Goldin, Claudia."America's Graduation from High School: The Evolution and Spread of Secondary Schooling in the Twentieth Century." Journal of Economic History 58 (June1998).

[16] Goldin, Claudia. "Egalitarianism and the returns to Education during the Great Transformation of American Education." Journal of Political Economy, Vol. 107, No. 6 pt.2. (1999)

[17] Goldin, Claudia. "The Rising (and the Declining) Significance of Gender." National Bureau of Economic Research Working Paper, No. 8915 (April 2002)

[18] Greenwood, Jeremy; Seshadri, Ananth; Yorukoglu, Mehmet."Engines of Liberation." Economie d'avant garde, Research Report No. 2, University of Rochester. (2001)

[19] Grossman, Gene M.; Helpman, Elhanan."Innovation and Growth in the Global Economy." Cambridge Massachusetts: MIT Press. (1993)

[20] Lebergott, Stanley. "Pursuing Happiness: American Consumers in the Twentieth Century." Princeton: Princeton University Press. (1993)

[21] Mincer, Jacob. "Labour Force Participation of Married Women: A Study of Labour Supply." In H. Gregg Lewis, ed., Aspects of Labor Economics. Universities-National Bureau Committee for Economic Research. Princeton, NJ: Princeton University Press. (1962)

[22] Romer, Paul. "The Origins of Endogenous Growth." Journal of Economic Perspectives, Vol. 8 No. 1. (Winter 1994)

[23] Rotella, Elyce J.. "The Transformation of the American Office: Changes in Employment and Technology." Journal of Economic History, Vol XLI, No. 1. (March 1981)

[24] Rotella, Elyce J.. "From Home to Office U.S. Women at Work 1870-1930." In R. Berkhofer, ed., Studies in American History and Culture No. 25. Michigan: UMI Research Press. (1981)

[25] Smith, James P.; Ward, Michael P. "Women's Wages and Work in the Twentieth Century." Rand Corporation, Santa Monica, CA.. (1984)

[26] U.S. Department of Commerce. Bureau of the Census. "Historical Statistics of the United States Colonial Times to 1970." Bicentennial Edition, Part 1. Washington, D.C. (1975)

[27] U.S. Department of Education. National Centre for Educational Statistics. "120 Years of American Education: A Statistical Portrait"edited by T. Snyder. Washington: Government Printing Office. (1993)